The Rydberg Constant Interpreted as the Gaussian Curvature, Gauss-Bohr-de Broglie Model – Two Shadow Projections of the Helix, Unlocking of the Fixed Constant c of the Speed of Light – New Tests for Old Physics

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ABSTRACT

We have newly interpreted the Rydberg constant R_∞ as the Gaussian curvature – the ratio of the 4π electron spin rotation to the area on the Gauss – Bohr sphere travelled by that electron. Rydberg constant for hydrogen R_H was newly derived and can be experimentally tested and compared with the value R_H derived from the reduced mass. The de Broglie electron on the helical path embedded on the Gauss – Bohr sphere was projected as two shadows: the real shadow $Re [\cos(t)]$ and the imaginary shadow $Im [i \sin(t)]$. This model differs from the Schrödinger famous quantum wave description in the physical interpretation. The wave amplitude is here interpreted as the distance of the shadow from the Gauss – Bohr sphere. Moreover, we have newly inserted into the wave equation curvature and torsion of that de Broglie helix. One very interesting result of this model is the estimation of the constant c of the speed of light with three additional significant figures. We have divided the very precise CODATA 2018 value for R_∞ expressed in frequency and the CODATA 1986 value for R_∞ expressed in wavenumber unit. Based on these precise spectroscopic data we might increase the accuracy of the constant c of the speed of light to twelve significant figures.

Keywords: The Rydberg constant – the Gaussian curvature, the Gauss – Bohr sphere, the Gauss – Bohr – de Broglie model, two shadows of the helix, the amplitude of shadows, curvature and torsion of de Broglie helix, unlocking of the fixed constant c of the speed of light, c with twelve significant figures.

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I. INTRODUCTION

The hydrogen atom has a central role in the development of modern physics. This simplest atom plays a key position in testing of fundamental theories and metrology. The precise spectroscopic data observed in the hydrogen spectra might even in the 21st century further deepen our knowledge about the events in the microworld, e.g., [1]–[10].

There are known many attempts to describe the events in the microworld by the classical physics or by the old quantum mechanics, e.g., [11]–[21]. All these ad-hoc models cannot compete with the mathematical description developed by the QM (quantum mechanics) in 1925, [22], [23]. We need to discover some new rules as our guides into the microworld.

We propose to interpret the Rydberg constant as the Gaussian curvature. This innovation enables to introduce a new description of events embedded on the surface of the Gauss – Bohr sphere without the electron jumps between fictitious orbitals. The electron helix embedded on the surface

of the Gauss – Bohr sphere could be observed as two shadows – the real shadow described as $Re [\cos(t)]$ and the imaginary shadow described as $Im [i \sin(t)]$.

We will formulate the wave equation for these shadows. In this model the amplitude of the wave is the distance of the shadow from the Gauss – Bohr sphere. The curvature and torsion of the de Broglie helix describe the motion of those shadows. In this model we might return back to old quantum model with a more realistic description of events in the microworld. This model offers a mathematical recipe more connected to the real events in the microworld.

There is one additional interesting result of this model: the estimation of the value of the constant c of the speed of light as the ratio of the Rydberg constant R_∞ expressed in the frequency units and the Rydberg constant R_∞ expressed in wavenumber units. Based on the very precise spectroscopic data we might achieve the value of the constant c with twelve significant figures.

II. THE RYDBERG CONSTANT INTERPRETED AS THE GAUSSIAN CURVATURE

In spectroscopy, the Rydberg constant, symbol R_∞ for heavy atoms or R_H for hydrogen, is the very important physical constant relating to the electromagnetic spectra of an atom. Rydberg constant is now one of the most accurately measured physical constants.

The Rydberg constant can be expressed as the combination of the fine-structure constant α and the Bohr radius a_0 as:

$$R_\infty = \frac{\alpha}{4\pi a_0} \quad (1)$$

Based on this relation, we have newly proposed the interpretation of the Rydberg constant as the Gaussian curvature:

$$4\pi R_\infty^2 = \frac{\alpha^2}{4\pi a_0^2} \quad (2)$$

The Rydberg constant is defined as the ratio of the 4π electron spin rotation divided by the size of the area on the Bohr – Gauss sphere around which the electron travelled.

The Gaussian curvature R_H of the Gauss – Bohr sphere at a point is the product of the principal curvatures κ_1 and κ_2 , at the given point (3):

$$R_H^2 = \kappa_1 \kappa_2 = R_\infty \sqrt{1 - \left(\frac{\lambda_e}{\alpha_0}\right)} R_\infty \sqrt{1 + \left(\frac{\lambda_e}{\alpha_0}\right)} = R_\infty^2 \sqrt{1 - \left(\frac{\lambda_e}{\alpha_0}\right)^2} \quad (3)$$

where λ_e is the Compton wavelength of the electron. Based on the relation between the fine-structure constant α , the Bohr radius a_0 and the λ_e :

$$2\pi\alpha = \frac{\lambda_e}{\alpha_0} \quad (4)$$

we can get the expression for the Gaussian curvature R_H as:

$$R_H = R_\infty \left(1 - (2\pi\alpha)^2\right)^{1/4} \quad (5)$$

Niels Bohr corrected the Rydberg constant for hydrogen R_H via the reduced mass as:

$$R_H = \frac{R_\infty}{1 + \frac{m_e}{m_H}} \quad (6)$$

where m_e is the electron mass, m_H is the proton mass, the ratio $m_H/m_e = 1\,836.152\,673\,43(11)$ based on CODATA 2018.

We can compare both predicted values R_H with the experimental data summarized in Table I.

TABLE I: COMPARISON OF THE RYDBERG CONSTANT R_H

CODATA 2018 (based on $c_{SI} = 299\,792\,458\, \text{m s}^{-1}$)
$R_\infty = 10\,973\,731.568\,160(21)\, \text{m}^{-1}$
R_H from the Lyman series experiment
$R_H = 10\,967\,882(\text{XX})\, \text{m}^{-1}$
R_H from the reduced mass correction
$R_H = 10\,967\,758\, \text{m}^{-1}$
R_H as the Gaussian curvature for hydrogen
$R_H = 10\,967\,959\, \text{m}^{-1}$

In order to evaluate both predicted values R_H with the experimental value R_H found from the Lyman series, we will need to improve the experimental value R_H on its seventh and eighth significant figures.

III. DE BROGLIE ELECTRON ON THE HELICAL PATH EMBEDDED ON THE GAUSS–BOHR SURFACE

Gaussian curvature is an intrinsic measure of curvature, depending only on distances that are measured on the surface, not on the way it is isometrically embedded in Euclidean space – this was expressed by Carl Fridrich Gauss in his Theorema egregium in 1827.

This Theorem enables to describe the events between the hydrogen electron and photons as de Broglie electron on the helical path embedded on the Gauss – Bohr sphere with the Bohr radius a_0 . In this model we can avoid “jumping” of electrons between fictitious orbitals.

We can describe the de Broglie electron helix embedded on the surface of the Gauss – Bohr sphere with parameters summarized in Table II.

TABLE II: PARAMETERS OF THE DE BROGLIE HELIX EMBEDDED ON THE GAUSS–BOHR SPHERE

Amplitude of the helix
$A = \frac{\lambda_{DE}}{4\pi n} = \frac{\lambda_e}{4\pi \alpha n} = \frac{a_0}{2n}$
Curvature of the helix
$\kappa = \frac{2\pi n}{\lambda_{DE}} = \frac{2\pi \alpha n}{\lambda_e} = \frac{n}{a_0}$
Torsion of the helix
$\tau = \frac{2\pi n}{\lambda_{DE}} = \frac{2\pi \alpha n}{\lambda_e} = \frac{n}{a_0}$
Speed of the electron on the Gauss – Bohr sphere
$V_n = \frac{V_{DE}}{n} = \frac{\alpha c}{n}$
Spin expressed as the angular momentum curvature
$S = m_e \frac{V_{DE}}{n} \frac{1}{\kappa} = m_e \frac{V_{DE}}{n} \frac{\lambda_{DE}}{2\pi n} = \frac{h}{2\pi} \frac{1}{n^2}$
Momentum of the electron
$p = \frac{h}{2\pi} \frac{1}{n^2} \kappa = \frac{h}{2\pi} \frac{1}{n^2} \frac{2\pi n}{\lambda_{DE}} = \frac{h}{\lambda_{DE}} \frac{1}{n}$
Energy of the electron
$E = \frac{h}{2\pi} \frac{1}{n^2} V_n \tau = \frac{h V_{DE}}{n^2}$

In Table II the index DE describes the de – Broglie waves. Niels Bohr postulated the stationary orbits at distances r for which the angular momentum of the revolving electron is an integer multiple of the reduced Planck constant:

$$m_e V r = n \frac{h}{2\pi} \quad (7)$$

We can express this condition for the de Broglie helix on the Gauss – Bohr sphere as:

$$m_e V_n a_0 = n \frac{h}{2\pi} \quad (8)$$

$$m_e c \frac{1}{n} \frac{V_{DE}}{c} \frac{\lambda_e}{2\pi \alpha} = n \frac{m_e V_{DE}}{n} \frac{\lambda_{DE}}{2\pi n} \quad (9)$$

$$\frac{h}{2\pi} \frac{1}{n} \alpha \frac{1}{\alpha} = n \frac{h}{2\pi} \frac{1}{n^2} \quad (10)$$

The interaction between the de Broglie electron with photons on the Gauss – Bohr sphere fulfills the Bohr postulate.

This model predicts one new experimental observable parameter. The electron travelling on the helical path embedded on the Gauss – Bohr sphere with radius a_0 should be found in the shell with the thickness Δr calculated from the principal quantum number n as:

$$\Delta r = a_0 \pm \frac{a_0}{2n} \quad (11)$$

IV. PROJECTIONS OF THE DE BROGLIE ELECTRON ON THE HELICAL PATH EMBEDDED ON THE GAUSS–BOHR SURFACE – TWO SHADOWS DESCRIBED BY THE WAVE MECHANICS

Erwin Schrödinger introduced the new quantum wave mechanics in 1925. This is our best model for the events in the microworld, however, this mathematical language is too abstract and raises several important questions. What is the nature of the wave? What is the amplitude of this wave? How to visualize the real effects behind these waves?

We have found a possible answer to these questions as the analogy with events in Plato's cave – we will guess the real effects from two shadows: one on the wall and one on the floor of the Plato's cave. The first shadow on the wall, we will describe as the real shadow as the function $\text{Re}[\cos(t)]$, the second shadow on the floor we will describe as the imaginary shadow as the function $\text{Im}[i \sin(t)]$. These two shadows are in *quadrature* with each other.

We will describe two projections of the de Broglie helix embedded on the surface of the Gauss – Bohr sphere as:

$$\Psi = \Psi_0 e^{i(\kappa x - V\tau)} = \frac{a_0}{2n} e^{i(\kappa x - V\tau)} \quad (12)$$

Here, the amplitude is expressed as the height of the shadow from the Gauss–Bohr sphere, curvature κ and torsion τ describe the evolution of that shadow:

$$\Psi = \frac{a_0}{2n} [\cos(\kappa x - V\tau) + i \sin(\kappa x - V\tau)] = \frac{a_0}{2n} \left[\cos\left(\frac{pn^2}{h}x - \frac{En^2}{h}t\right) + i \sin\left(\frac{pn^2}{h}x - \frac{En^2}{h}t\right) \right]$$

This model predicts that the mathematical system of the quantum mechanics tries to describe the real effects from the shadows. This is the reason why the interpretation of quantum wave mechanics might cause such difficulties. How exactly can we guess the real situation from those shadows?

V. THE HIGH ACCURACY OF THE RYDBERG CONSTANT AND THE UNLOCKING OF THE FIXED CONSTANT C OF THE SPEED OF LIGHT

We have analyzed the historical improvement in the accuracy of the Rydberg constant and have noticed an anomaly in those data around the year 1986, e.g., [24]–[33].

The best experimental techniques for the measurement of the Rydberg constant before the year 1986 were measurements in wavenumber units.

In the begin of 1990's the experimental techniques measuring the Rydberg constant in frequencies significantly improved their accuracy and now the Rydberg constant measured in frequencies achieved the highest level among the physical constants. In 1983 the value of the constant of the speed of light was fixed, after that the value R_∞ in wavenumber units is converted from R_∞ measured in frequency via this fixed constant c_{SI} .

We propose to express the value of constant of the speed of light c_{RY} as the ratio of R_∞ in frequencies to R_∞ in wavenumbers:

$$c_{RY} = \frac{R_\infty(\text{CODATA 2018}) \left[\frac{Hz}{m^{-1}} \right]}{R_\infty(\text{CODATA 1986}) \left[\frac{Hz}{m^{-1}} \right]} = \frac{3.289\,841\,960\,2508(64) * 10^{15} Hz}{10\,973\,731.534(13) m^{-1}} = 299\,792\,458.933(XX) m s^{-1}$$

We can compare this value c_{RY} with the last experimental values of c before that this constant was fixed in 1983.

TABLE III: THE EXPERIMENTAL VALUES OF THE SPEED OF LIGHT

1972 Evenson [34]	29979245(8,8)(1.2)(peak)
1973 CCDM	29979245(8,0)(1.2)
1974 Blaney [35]	29979245(9,0)(0.6)
1978 Woods [36]	29979245(8,8)(0.2)
1983 BIPM SI fixed	29979245(8)
2021 Stávek [37]	29979245(8,9,1,5)(0.0XX)
2021 This paper	29979245(8,9,3,3)(0.0XX)

We propose to further develop spectroscopic techniques for the measurement of the Rydberg constant in wavenumber units. We have now a very good opportunity to achieve the accuracy of the constant of the speed of light with twelve significant figures. The fixing of SI constants might mask the real effects [38].

VI. CONCLUSION

The hydrogen atom and the precise hydrogen spectroscopy might guide us to the microworld secrets even in the 21st century. The extremely precise accuracy of the Rydberg constant both R_∞ and R_H could support us in the further search for the interpretation of our mathematical models. Did we achieve the ultimate knowledge about Nature? How to distinguish between shadows and the real effects?

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