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RESEARCH ARTICLE

A Newly-Derived Cosmological Redshift Formula Which Solves the Hubble Tension and Yet Maintains Consistency with $T_t = T_0(1+z)$, the $R_h = ct$ Principle and the Stefan-Boltzmann Law

Espen Gaarder Haug[1,*] and Eugene Terry Tatum[2]

ABSTRACT

Numerous cosmological redshift formulae have been suggested in the field of cosmology. One of these is the well-known cosmological redshift formula used in the Λ-CDM model and in some $R_h = ct$ models. In a recent type of "black hole" cosmology model, the redshift formula is derived from three fundamental principles: that the Stefan-Boltzmann law holds with respect to perfect and almost perfect black bodies; that the universe follows the $R_h = ct$ principle; and that the time-dependent CMB temperature in relation to cosmological redshift is given by the observed relation $T_t = T_0(1+z)$. These three principles have recently been used by Haug and Tatum [1] to derive $z = \sqrt{\frac{R_h}{R_t}} - 1$, which has led to a simple but powerful solution to the Hubble tension [2]–[4]. This note highlights the importance of these three principles in relation to the new cosmological redshift formula and describes new ways to represent the same cosmological redshift formula.

Keywords: Black body, black hole cosmology, Cosmological redshift, R_h= *ct* cosmology.

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[1] *Norwegian University of Life Sciences and Tempus Gravitational Laboratory, Norway.*
[2] *Independent Researcher, Bowling Green, Kentucky, USA.*

[*] ***Corresponding Author:***
e-mail: espenhaug@mac.com

1. How to Derive the New Cosmological Redshift Formula

First, we will briefly explain how this redshift formula was derived. The following two formulae are based on a 2015 publication by Tatum *et al.* [5], which suggested:

$$T_0 = \frac{\hbar c}{k_b 4\pi \sqrt{R_h 2 l_p}} \tag{1}$$

If we assume that the $R_h = ct$ cosmological principle holds true, we must have:

$$T_t = \frac{\hbar c}{k_b 4\pi \sqrt{R_t 2 l_p}} \tag{2}$$

Subsequently, Haug and Wojnow [6], [7] proved that the same formulae could be derived from the Stefan-Boltzmann law. To understand the significance of this, one must first be aware that the cosmic microwave background (CMB) radiation spectrum is that of a nearly perfect black body, as pointed out by Muller *et al.* [8], who stated:

> *"Observations with the COBE satellite have demonstrated that the CMB corresponds to a nearly perfect black body, characterized by a temperature T_0 at $z = 0$, which is measured with very high accuracy, $T_0 = 2.72548 \pm 0.00057$ K."*

This fact is also implied by the recent work of Dhal and Paul [9]. In addition to the Stefan-Boltzmann law and the $R_h = ct$ principle, we have the observed relation between the current observed CMB temperature and the CMB temperature at an earlier cosmic time, as a function of the observed redshift:

$$T_t = T_0(1+z) \tag{3}$$

As pointed out by Riechers *et al*. [10]:

> *"No deviations from the expected* $(1+z)$ *scaling behaviour of the microwave background temperature have been seen, but the measurements have not extended deeply into the matter-dominated era of the Universe at redshifts* $z > 3.3$*."*

Now, if we replace T_0 and T_t with the Stefan-Boltzmann-derived formulae for these, we obtain:

$$\frac{\hbar c}{k_b 4\pi\sqrt{R_t 2l_p}} = \frac{\hbar c}{k_b 4\pi\sqrt{R_h 2l_p}}(1+z) \tag{4}$$

That, when solved for z, gives:

$$z = \sqrt{\frac{R_h}{R_t}} - 1 \tag{5}$$

This is the Haug and Tatum cosmological redshift equation, fully derived based on three important principles:

1. The Stefan-Boltzmann law must hold for nearly perfect black bodies. As we have noted, the cosmic microwave background is considered the most nearly perfect black body radiation spectrum observed, so this principle must apply.
2. The observed relation $T_t = T_0(1+z)$ must hold. All evidence points in this direction, even though, based on observations alone, one cannot entirely rule out the possibility of $T_t = T_0(1+z)^{1-\beta}$ with β slightly different from zero, see [10], [11]. However, the probability of β being slightly different from zero is very low, as this would lead to a significantly more complicated model and potential inconsistencies. At present, it appears to be highly probable that the relationship $T_t = T_0(1+z)$ is correct and that any serious cosmological model must be consistent with this.
3. The final principle required to derive our cosmological redshift equation is our assumption of $R_h = ct$ cosmology. A series of observational studies appears to favor $R_h = ct$ cosmology. However, caution is needed here, since the Melia-type $R_h = ct$ cosmology differs in several significant ways from the Haug-Tatum $R_h = ct$ cosmology (HTC). One major difference is the redshift formula derived above and the fact that HTC can accurately predict the current CMB temperature.

If the $R_h = ct$ principle holds true (and much points in that direction), then the evidence in favor of the $z = \sqrt{\frac{R_h}{R_t}} - 1$ cosmological redshift formula is strong. One can potentially solve the Hubble tension using the assumed $z = \frac{R_h}{R_t} - 1$ or even $z = (\frac{R_h}{R_t})^x - 1$ (see [4]), but the problem with such models then becomes that they are inconsistent with $T_t = T_0(1+z)$ for any x different than $\frac{1}{2}$. Or one can still potentially make it consistent with $T_t = T_0(1+z)$ for $x \neq \frac{1}{2}$, but then one cannot make such a model consistent with the observed and Stefan-Boltzmann law-derived current CMB temperature.

2. Different Ways to Re-Write the New Cosmological Redshift Formula

It is also worth noting that our cosmological redshift formula can be trivially rewritten in multiple forms, (see [12]):

$$z = \sqrt{\frac{R_h}{R_t}} - 1 = \sqrt{\frac{H_t}{H_0}} - 1 = \sqrt{\frac{t_{h,0}}{t_{h,t}}} - 1 = \sqrt{\frac{M_c}{M_{c,t}}} - 1 = \frac{T_t}{T_0} - 1 \tag{6}$$

where $t_h = \frac{1}{H_0}$, $t_t = \frac{1}{H_t}$, $M_c = \frac{c^2 R_h}{2G}$, $M_{c,t} = \frac{c^2 R_t}{2G}$. Note that the cosmological redshift term z, as function of CMB temperature, is not squared. The reason for this is that the only variable in T_t is already related to $\sqrt{R_t}$. This relationship comes directly from the Stefan-Boltzmann law. The equalities in (6) are a direct result of our assumption of $R_h = ct$ cosmology (a linear model), meaning that the percentage change in R_t, H_t, t_t, $M_{c,t}$ is the same as the percentage change in R_t.

We can even write the cosmological redshift as a function of critical energy density over time. We must have $z = \left(\frac{\rho_c}{\rho_{c,t}}\right)^{\frac{1}{4}} - 1$ as $\rho_c = \frac{3c^4}{R_h^2 8\pi G}$ and $\rho_{c,t} = \frac{3c^4}{R_t^2 8\pi G}$. Here the scaling is different as the energy density is a function of R_t^2 and not simply of R_t as is the case for H_t, t_t and $M_{c,t}$.

Since we model the universe as a growing black hole, we can even link the cosmological redshift to the Bekenstein-Hawking [13]–[16] entropy. We obtain:

$$z = \left(\frac{S_{BH}}{S_{BH,t}}\right)^{\frac{1}{4}} - 1 \tag{7}$$

This is no surprise, since the entropy of a black hole is given by $S_{BH} = \frac{A}{4l_p^2} = \frac{4\pi R_h^2}{4l_p^2}$, and the Bekenstein-Hawking entropy at earlier epochs must be $S_{BH,t} = \frac{A}{4l_p^2} = \frac{4\pi R_t^2}{4l_p^2}$, where $R_t = ct$. Thus, (7) can easily be seen to be consistent with $z = \sqrt{\frac{R_h}{R_t}} - 1$.

3. Conclusion

In conclusion, the Haug and Tatum cosmological redshift is the only cosmological redshift derived according to the following three principles: the Stefan-Boltzmann law; the observed $T_t = T_0(1 + z)$ relation; and the $R_h = ct$ principle. We cannot see how the different Melia [17] $R_h = ct$ cosmological model redshift formula can be consistent with all three principles simultaneously, but we are open to hearing other opinions on this topic. Despite this possible weakness in the Melia model, Melia [18], [19] has done impressive and extensive research demonstrating how the $R_h = ct$ principle appears to be favored by observations, when compared to the Λ-CDM model. We believe that different variants of $R_h = ct$ cosmology models deserve increased attention and comparison in future studies.

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