European Journal of Applied Physics
Vol 7 | Issue 1 | January 2025
ISSN 2684-4451

RESEARCH ARTICLE

Length Contraction, Time Dilation, Mass, Momentum and Energy Equations, Particle and Antiparticle Potential Energy, Pair Production and Annihilation Energy Equations from Harmonic Oscillator Rest Energy Equation and New Relations from Uncertainty Principle

Abdürrezzak Emin Bozdoğan*

ABSTRACT

The harmonic oscillator total rest energy equation is derived from simple and quantum-mechanical harmonic oscillator equations. Length contraction, time dilation, relativistic mass, momentum, and energy equations for a particle, electron, and Planck particle are derived from harmonic oscillator total rest energy equation and new relations derived from the Heisenberg uncertainty principle. Particle and antiparticle, electron and positron potential energy equations; particle-antiparticle, electron-positron pair productions, and pair annihilations minimum energy equations are derived from harmonic oscillator total rest energy equation. The rest energy equation is derived from a nonrelativistic differential equation.

Keywords: Harmonic oscillator total rest energy equation, new relations from uncertainty principle, pair production and annihilation potential energy equations, relativistic equations.

Submitted: October 30, 2024
Published: January 02, 2025

10.24018/ejphysics.2025.7.1.352

Department of Chemistry, Faculty of Arts & Science, Yildiz Technical University, Turkey.

****Corresponding Author:***
e-mail: abdurrezzakbozdogan@gmail.com

1. Introduction

The electron Compton radius or the reduced Compton wavelength, which is obtained from scattering experiments and is taken as the photon scattering radius of the electron is given by [1]–[3]:

$$r_C m_e c = \hbar \tag{1}$$

where $r_C = \lambda_C/2\pi$ is the electron Compton radius, λ_C is the Compton wavelength, m_e is the electron rest mass, c is the speed of light in vacuum and $\hbar$ is the reduced Planck constant. From (1), (2) is written:

$$m_e c^2 = h f_C \tag{2}$$

where h is the Planck constant and f_C is the Compton frequency.

Van Belle [2], [4] proposed a two-dimensional harmonic oscillator model for zitterbewegung electron, and assumed that the free electron is a pointlike charge in an electromagnetic orbital oscillation of radius a and rotates at tangential speed of $c = \omega a$. He derived the total energy equation for each oscillator as:

$$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2 a^2 = \frac{1}{2}mc^2 \tag{3}$$

where $\omega = (k/m)^{1/2}$ is the angular frequency, m is the electron mass and k is the force constant.

Daywitt [3] obtained (4) from the Planck mass $m_* = \sqrt{\hbar c/G}$ and the Planck length $r_* = \sqrt{\hbar G/c^3}$,

$$r_* m_* c = \hbar \tag{4}$$

and derived the ground level Planck particle quantum harmonic oscillator total energy equation by using (4),

$$\frac{1}{2}m_* v_x^2 + \frac{1}{2}K_* x^2 = \frac{1}{2}m_* \omega_*^2 r_*^2 = \frac{1}{2}m_* c^2 = \frac{1}{2}\hbar\omega_* \tag{5}$$

where $\omega_* = (K_*/m_*)^{1/2} = c/r_* = 1/t_*, K_*$ is the spring constant, $t_* = \sqrt{\hbar G/c^5}$ is the Planck time [5], [6].

In Section 2, Van Belle [4] and Daywitt [3] equations for electron and Planck particle harmonic oscillators are generalized and harmonic oscillator total rest energy equation is derived from simple and quantum-mechanical harmonic oscillators equations. Length contraction, time dilation, relativistic mass, momentum and energy equations for a particle, electron and Planck particle are derived from harmonic oscillator total rest energy equation and new relations derived from Heisenberg uncertainty principle. In Section 3, particle and antiparticle, electron and positron potential energy equations; particle-antiparticle, electron-positron pair productions and pair annihilations minimum energy equations are derived from harmonic oscillator total rest energy equation. In Section 4, rest energy equation is derived from nonrelativistic differential equation.

2. Derivations of Length Contraction, Time Dilation, Relativistic Mass, Momentum and Energy Equations from Harmonic Oscillator Total Rest Energy Equation and New Relations from Heisenberg Uncertainty Principle

Total mechanical energy in simple harmonic motion is given by [1, p. 446]:

$$\frac{1}{2}mv_x^2 + \frac{1}{2}k' x^2 = \frac{1}{2}k' A^2 = \text{Constant} \tag{6}$$

where x is the displacement from equilibrium, A is the maximum displacement or the amplitude of the motion, k' is the force constant and m is the particle mass. Since the motion is one-dimentional, $v^2 = v_x^2$. When the particle reaches the point $x = A$, it comes momentarily to rest before reversing direction. That is, when $x = A$ (or $-A$), $v_x = 0$. At this rest point the energy is entirely potential and constant $1/2k' A^2$, and is equal to the rest energy $1/2mc^2$. From these expressions, (7) can be written as:

$$\frac{1}{2}k' A^2 = \frac{1}{2}mc^2 \tag{7}$$

The relationship between the total energies of the simple and the ground level quantum-mechanical harmonic oscillators is given by [1, p. 1353]:

$$\frac{1}{2}k' A^2 = \frac{1}{2}\hbar\omega \tag{8}$$

where $\omega = \left(k'/m\right)^{1/2} = 2\pi f$ and f is frequency. Equation (8) shows that the energy of the photon is not equal to the energy of the ground level, but rather it is the energy difference between two levels. From (6)–(8), harmonic oscillator total rest energy eqution is obtained.

$$\frac{1}{2}mv_x^2 + \frac{1}{2}k' x^2 = \frac{1}{2}k' A^2 = \frac{1}{2}\hbar\omega = \frac{1}{2}mc^2 \tag{9}$$

When the particle is at its maximum displacement ($x = \pm A$) and instantaneously at rest, $v_x = 0$. When the particle is at equilibrium ($x = 0$) and moving at its maximum speed, $v_x = v_{max} = c$, and the maximum momentum of the particle is $p_{max} = mv_{max} = mc$, where m is the rest mass [1, p.1354].

Heisenberg uncertainty principal for position and momentum is defined as [1, p.1354].

$$\Delta x \Delta p_x = \frac{\hbar}{2} \tag{10}$$

The uncertainties in the particle's position and momentum (calculated as standart deviations) for harmonic oscillator are given $\Delta x = A/\sqrt{2}$ and $\Delta p_x = p_{max}/\sqrt{2}$, respectively, [1, p.1354]. When these uncertainties are substituted in (10), (11) is obtained.

$$Amc = \hbar \tag{11}$$

From (9), (12) is obtained.

$$x = A\sqrt{1-\frac{v_x^2}{c^2}} \tag{12}$$

where $A = c\sqrt{m/k'}$. The positive square root of x is taken since x must be positive. When $x = l$, $A = l_0$ and $v_x = v$ are substituted in (12), length contraction is obtained [1, p. 1234]:

$$l = l_0\sqrt{1-\frac{v^2}{c^2}} \tag{13}$$

When (12) is substituted in (11), (14) is obtained:

$$Amc = x\frac{m}{\sqrt{1-\frac{v^2}{c^2}}}c = x\,m_{rel}c = \hbar \tag{14}$$

where $v_x = v$ and m_{rel} is the relativistic mass [1, p. 1244]:

$$m_{rel} = \frac{m}{\sqrt{1-\frac{v^2}{c^2}}} \tag{15}$$

When (15) is multiplied by v, relativistic momentum p is obtained [1, p. 1243]:

$$p = m_{rel}v = \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}} \tag{16}$$

When (14) is multiplied by c, (17) is obtained:

$$Amc^2 = x\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} = xE = \hbar c \tag{17}$$

where E is the total energy [1, p. 1247]:

$$E = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} \tag{18}$$

When $v = 0$ is substituted in (18), rest energy E_0 is obtained:

$$E_0 = mc^2 \tag{19}$$

From (18) and (19), relativistic kinetic energy K is obtained:

$$K = E - E_0 = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} - mc^2 \tag{20}$$

From (1), (4) and (11), the relation can be written as follows:

$$r_C m_e c = r_* m_* c = A\,mc = \hbar \tag{21}$$

When $m = m_e$ is substituted in (21), $A = r_C$ is obtained. When $A = r_C$ and $x = x_C$ are substituted in (12), contraction of Compton radius [7] is obtained:

$$x_C = r_C\sqrt{1-\frac{v^2}{c^2}} \tag{22}$$

When (22) is substituted in (1), the relation is obtained for electron:

$$r_C m_e c = x_C\frac{m_e}{\sqrt{1-\frac{v^2}{c^2}}}c = x_C m_{e_{rel}} c = \hbar \tag{23}$$

where $m_{e_{rel}}$ is the relativistic mass of electron:

$$m_{e_{rel}} = \frac{m_e}{\sqrt{1-\frac{v^2}{c^2}}} \tag{24}$$

When (24) is multiplied by v, relativistic momentum p_e of electron is obtained:

$$p_e = m_{e_{rel}} v = \frac{m_e v}{\sqrt{1-\frac{v^2}{c^2}}} \tag{25}$$

When (23) is multiplied by c, (26) is obtained:

$$r_C m_e c^2 = x_C \frac{m_e c^2}{\sqrt{1-\frac{v^2}{c^2}}} = x_C E_e = \hbar c \tag{26}$$

where E_e is the total energy of electron.

$$E_e = \frac{m_e c^2}{\sqrt{1-\frac{v^2}{c^2}}} \tag{27}$$

When $v = 0$ is substituted in (27), rest energy E_{e_0} of electron is obtained:

$$E_{e_0} = m_e c^2 \tag{28}$$

Relativistic kinetic energy of electron K_e is obtained from (27) and (28):

$$K_e = \frac{m_e c^2}{\sqrt{1-\frac{v^2}{c^2}}} - m_e c^2 \tag{29}$$

When $m = m_*$ is substituted in (21), $A = r_*$ is obtained. When $A = r_*$ and $x = x_*$ are substituted in (12), contraction of Planck length is obtained:

$$x_* = r_* \sqrt{1-\frac{v^2}{c^2}} \tag{30}$$

When (30) is substituted in (4), the relation is obtained for Planck particle:

$$r_* m_* c = x_* \frac{m_*}{\sqrt{1-\frac{v^2}{c^2}}} c = x_* m_{*_{rel}} c = \hbar \tag{31}$$

where $m_{*_{rel}}$ is the relativistic mass of Planck particle.

$$m_{*_{rel}} = \frac{m_*}{\sqrt{1-\frac{v^2}{c^2}}} \tag{32}$$

When (32) is multiplied by v, relativistic momentum p_* of Planck particle is obtained:

$$p_* = m_{*_{rel}} v = \frac{m_* v}{\sqrt{1-\frac{v^2}{c^2}}} \tag{33}$$

When (31) is multiplied by c, (34) is obtained:

$$r_* m_* c^2 = x_* \frac{m_* c^2}{\sqrt{1-\frac{v^2}{c^2}}} = x_* E_* = \hbar c \tag{34}$$

where E_* is the total energy of Planck particle:

$$E_* = \frac{m_* c^2}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{35}$$

When $v = 0$ is substituted in (35), rest energy E_{*0} of Planck particle is obtained.

$$E_{*0} = m_* c^2 \tag{36}$$

Relativistic kinetic energy of Planck particle K_* is obtained from (35) and (36):

$$K_* = \frac{m_* c^2}{\sqrt{1 - \frac{v^2}{c^2}}} - m_* c^2 \tag{37}$$

From r_* and t_*, (38) is obtained:

$$r_* t_* = \frac{G}{c^4}\hbar \tag{38}$$

From (21) and (38)–(40) are obtained:

$$r_* t_* = r_* \frac{m_* G}{c^3} = r_C \frac{m_e G}{c^3} = A \frac{mG}{c^3} = \frac{G}{c^4}\hbar \tag{39}$$

$$r_* t_* = r_C t_C = A t_0 = \frac{G}{c^4}\hbar \tag{40}$$

where $t_* = m_* G/c^3$, $t_C = m_e G/c^3$ and $t_0 = mG/c^3$ are the Planck, electron and particle times. When (12) is substituted in (40), (41) is obtained:

$$A t_0 = x \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}} = xt = \frac{G}{c^4}\hbar \tag{41}$$

where t is time dilation for particle [1, p. 1229], [8, p. 209]:

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{42}$$

When (22) is substituted in (40), (43) is obtained:

$$x_C \frac{t_C}{\sqrt{1 - \frac{v^2}{c^2}}} = x_C t_{C_{rel}} = \frac{G}{c^4}\hbar \tag{43}$$

where $t_{C_{rel}}$ is time dilation for electron:

$$t_{C_{rel}} = \frac{t_C}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{44}$$

When (30) is substituted in (40), (45) is obtained:

$$x_* \frac{t_*}{\sqrt{1 - \frac{v^2}{c^2}}} = x_* t_{*_{rel}} = \frac{G}{c^4}\hbar \tag{45}$$

where $t_{*_{rel}}$ is time dilation for Planck particle:

$$t_{*_{rel}} = \frac{t_*}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{46}$$

3. Derivations of Particle and Antiparticle, Electron and Positron Potential Energy Equations, Particle-Antiparticle, Electron-Positron Pair Productions and Pair Annihilations Minimum Energy Equations from Harmonic Oscillator Total Rest Energy Equation

Particle and antiparticle can be treated as harmonic oscillators. Potential energy equation for a particle can be written from (9) as:

$$2U = k^{\cdot}x^2 = mc^2 - mv^2 \tag{47}$$

where $x = 0$, $2U = 0$, and $v = c$. When x increases, v decreases [1, p.1353] and $2U$ increases. When $x = A$, $v = 0$, $2U = 2U_{max}$, and (48) can be written from (47).

$$2U_{max} = k^{\cdot}A^2 = mc^2 \tag{48}$$

From (47), potential energy equation for electron can be written as:

$$2U_{e^-} = k_e^{'}x^2 = m_e c^2 - m_e v^2 \tag{49}$$

When $x = 0$, $2U_{e^-} = 0$, and $v = c$. When x increases, v decreases and $2U_{e^-}$ increases. When $x = A = r_C$, $v = 0$, $2U_{e^-} = 2U_{e^-_{max}}$, and (50) can be written from (49):

$$2U_{e^-_{max}} = k_e^{'}r_C^2 = m_e c^2 \tag{50}$$

According to the symmetry property of (47), potential energy equation for an antiparticle can be written as:

$$-2U = -k^{\cdot}x^2 = -mc^2 + mv^2 \tag{51}$$

When $x = 0$, $-2U = 0$, and $v = c$. When x increases, v decreases and $-2U$ increases. When $x = A$, $v = 0$, $-2U = -2U_{max}$, and (52) can be written from (51):

$$-2U_{max} = -k^{\cdot}A^2 = -mc^2 \tag{52}$$

The minus sign shows the opposite direction of antiparticle to particle. From (51), potential energy equation for positron can be written as:

$$2U_{e^+} = -2U_{e^-} = -k_e^{'}x^2 = -m_e c^2 + m_e v^2 \tag{53}$$

When $x = 0$, $-2U_{e^-} = 0$, and $v = c$. When x increases, v decreases and $-2U_{e^-}$ increases. When $x = A = r_C$, $v = 0$, $-2U_{e^-} = -2U_{e^-_{max}}$, and (54) can be written from (53):

$$2U_{e^+_{max}} = -2U_{e^-_{max}} = -k_e^{'}r_C^2 = -m_e c^2 \tag{54}$$

Equations (53) and (54) show that positron is a real particle and not a hole or vacancy [1, p.1482], [9], and it is not an electron traveling backward in time [10].

The minimum energy for particle-antiparticle pair production is obtained from the difference between (48) and (52) as:

$$4U_{max} = 2k^{'}A^2 = 2mc^2 \tag{55}$$

The minimum energy for electron-positron pair production is obtained from the difference between (50) and (54) as:

$$2U_{e^-_{max}} - 2U_{e^+_{max}} = 4U_{e^-_{max}} = 2k_e^{'}r_C^2 = 2m_e c^2 \tag{56}$$

Fig. 1 shows the potential energies for electron e_- and positron e_+ as functions of position x. The curves for electron and positron are symmetric about the x-axis and opposite directions due to the energy and momentum conservations.

From (8), (48) and (52), (57) and (58) can be written as:

$$2U_{max} = mc^2 = hf = pc \tag{57}$$

$$-2U_{max} = -mc^2 = -hf = -pc \tag{58}$$

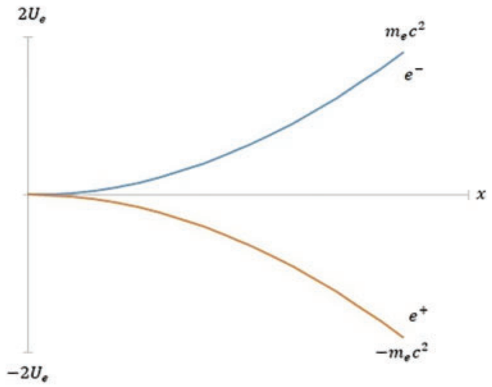

Fig. 1. Potential energies $2U_e$ and $-2U_e$ for electron e^- and positron e^+ as functions of position x.

where p is the photon momentum. The energy produced by pair annihilation is obtained from the difference between (57) and (58) as:

$$4U_{max} = 2mc^2 = 2hf = 2pc \tag{59}$$

From (57) and (58), (60) and (61) can be written for electron and positron as:

$$2U_{e^-max} = m_ec^2 = hf_C = p_Cc \tag{60}$$

$$2U_{e^+_{max}} = -2U_{e^-max} = -m_ec^2 = -hf_C = -p_Cc \tag{61}$$

where p_C is the Compton momentum. The energy produced by the electron-positron pair annihilation is obtained from the difference between (60) and (61) as:

$$2U_{e^-_{max}} - 2U_{e^+_{max}} = 4U_{e^-_{max}} = 2m_ec^2 = 2hf_C = 2p_Cc \tag{62}$$

Two photons have equal and opposite momenta by momentum conservation and have equal energies.

4. Derivation of Rest Energy from Differential Equation

Einstein derivations of rest energy are based on the radiation pressure [8, p.232–234] and special relativity [11, p.49,50]. In this study, rest energy is derived from differential equation.

For nonrelativistic speed ($v \ll c$), the rate of change of total energy E with velocity v can be given as:

$$\frac{dE}{dv} = aEv \tag{63}$$

From (63), (64) can be written:

$$\int_{E_0}^{E} \frac{dE}{E} = a\int_0^v vdv \tag{64}$$

where $E = E_0 + K$ is the total energy, E_0 is the rest energy, $K = mv^2/2$ is the kinetic energy m is the rest mass and a is a constant. If $a = 1/c^2$ is substituted in (64), (65) is obtained:

$$\frac{E}{E_0} = e^{\frac{v^2}{2c^2}} \tag{65}$$

From (65), (66) can be written:

$$\frac{E_0 + K}{E_0} = 1 + \frac{v^2}{2c^2} + \dots \tag{66}$$

When $K = mv^2/2$ is substituted in (66), the rest energy is obtained:

$$E_o = mc^2 \tag{67}$$

5. Conclusion

The harmonic oscillator total rest energy equation was derived from simple and quantum-mechanical harmonic oscillator equations. New relations were derived from Heisenberg's uncertainty principle. Length contraction, time dilation, relativistic mass, momentum, and energy equations for a particle, electron, and Planck particle were derived from harmonic oscillator total rest energy equation and new relations. Particle and antiparticle, electron and positron potential energy equations; particle-antiparticle, electron-positron pair productions, and pair annihilations energy equations were derived from harmonic oscillator total rest energy equation. The rest energy equation was derived from a nonrelativistic differential equation.

Conflict of Interest

The author declares that there is no conflict of interest.

References

[1] Young HD, Freedman RA. *University Physics*. 13th ed. Boston, New York, Cape Town, San Francisco, Hong Kong, London, Madrid, Mexico City, Montreal, Munich, Paris, Singapore, Sidney, Tokyo, Toronto: Addison-Wesley; 2012.

[2] Santos IU. The zitterbewegung electron puzzle. *Phys Essays*. 2023;36(3):299–335.

[3] Daywitt WC. The Planck vacuum. *Prog Phys*. 2009;1:20–6.

[4] Van Belle JL. The zitterbewegung interpretation of quantum mechanics. Mar 2019. Available from: https://vixra.org/pdf/1901.0105vD.pdf.

[5] Daywitt WC. Zero-point oscillations in the Planck-vacuum state and its coordinate uncertainty. *Eur J Eng Res Sci*. 2021;6(4):13–5.

[6] Daywitt WC. The Heisenberg uncertainty principle in the Planck vacuum theory. *Eur J Appl Phys. September*. 2022;4(5):1–3.

[7] Di Tommaso AO, Vassallo J. Electron structure, ultra-dense hydrogene and low energy nuclear reactions. *J Condens Matter Nucl Sci*. 2019;29:525–47.

[8] Born M. *Einstein's Theory of Relativity*. Third ed. New York: E.P. Dutton and Company; 1922.

[9] Dirac PAM. Theory of electrons and positrons. *Nobel Lecture*. Dec 12, 1933.

[10] Feynman RP. The theory of positrons. *Phys Rev*. 1949;76(6):749–59.

[11] Einstein A. *The Meaning of Relativity*. Princeton: Princeton University Press; 1921.