Analytical Solutions of the Schrödinger Equation with Class of Yukawa Potential for a Quarkonium System Via Series Expansion Method

E. P. Inyang, E. P. Inyang, I. O. Akpan, J. E. Ntibi and E. S. William

ABSTRACT

A class of Yukawa potential is adopted as the quark-antiquark interaction potential for studying the mass spectra of heavy mesons. The potential was made to be temperature-dependent by replacing the screening parameter with Debye mass. We solved the radial Schrödinger equation analytically using the series expansion method and obtained the energy eigenvalues. The present results are applied for calculating the mass spectra of heavy mesons such as charmonium $c\bar{c}$ and bottomonium $b\bar{b}$. Two special cases were considered when some of the potential parameters were set to zero, resulting into Hellmann potential, and Coulomb potential, respectively. The present potential provides satisfying results in comparison with experimental data and the work of other researchers.

Keywords: Schrödinger equation, series expansion method, class of Yukawa potential, heavy mesons.

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I. INTRODUCTION

The studies of heavy quarkonium systems such as charmonium and bottomonium have a vital role for understanding the quantitative tests of quantum chromodynamics (QCD) and the standard model [1]. These systems can be studied within the Schrödinger equation (SE) [2]. The solution of SE with spherically symmetric potential is one of the important problems in physics and chemistry. It plays a large vital role for spectroscopy, atoms, molecules, and nuclei, in particular, the properties of constituent's particles and dynamics of their interactions [3]. There potential should take into account the two important features of the strong interaction, namely, asymptotic freedom and quark confinement [4]-[9]. The SE has been solved using various methods such as, asymptotic iteration method (AIM) [2], [10] Laplace transformation method [11], super symmetric quantum mechanics method (SUSQM) [12], Nikiforov-Uvarov (NU) method [13]-[27], series expansion method [28] and others.

The most fundamental potential used in studying quarkonium system is the Cornell potential, also known as Killingbeck potential. Most researchers have carried out works with Cornell potential. For instance, Vega and Flores [29] solved the Schrödinger equation with the Cornell potential using the variational method and super symmetric quantum mechanics (SUSYQM). Ciftci and Kisoglu [30] addressed non-relativistic arbitrary l states of quark-

antiquark through the Asymptotic Iteration Method (AIM). The energy eigenvalues with any $l \neq 0$ states and mass of the massive quark-antiquark system (quarkonium) were gotten. An analytic solution of the N-dimensional radial Schrödinger equation with the mixture of vector and scalar potentials via the Laplace transformation method (LTM) was studied by [31]. Their results were employed to analyze the different properties of the heavy-light mesons. Al-Jamel and Widyan [32] studied heavy quarkonium ($c\bar{c}$ and $b\bar{b}$) mass spectra in a Coulomb field plus quadratic potential using the Nikiforov-Uvarov method. In their work, the spin-averaged mass spectra of heavy quarkonia ($c\bar{c}$ and $b\bar{b}$) in a Coulomb plus quadratic potential is analyzed within the non-relativistic Schrödinger equation. Al-Oun et al. [33] examine heavy quarkonia ($c\bar{c}$, and $b\bar{b}$) characteristics in the general framework of a non-relativistic potential model consisting of a Coulomb plus quadratic potential. Furthermore, Omugbe et al. [34] solved the SE with Killingbeck potential plus an inversely quadratic potential model. They obtained the energy eigenvalues and the mass spectra of the heavy and heavy-light meson systems. In addition, Cari et al. [35] studied the energy spectra of mesons and hadronic interactions using Numerov's method. Their solutions were used to describe the phenomenological interactions between the charm-anticharm quarks via the model. The model accurately predicts the mass spectra of charmed quarkonium as an example of mesonic systems. Inyang et al. [36] obtained the Klein-Gordon equation

solutions for the Yukawa potential using the Nikiforov-Uvarov method. The energy eigenvalues were obtained both in relativistic and non-relativistic regime. They applied the results to calculate heavy-meson masses of charmonium $c\bar{c}$ and bottomonium $b\bar{b}$.

The class of Yukawa potential is greatly important with applications, cutting across nuclear physics and condensed matter physics [37]. It takes the form:

$$V(r) = -\frac{ar + bre^{-\delta r} - ce^{-2\delta r}}{r^2} \quad (1)$$

where a , b , and c are potential strength, δ is the screening parameter which control the shape of the potential as shown in Fig. 1. It can be deduced that when $c = 0$, Eq. (1) reduces to Hellmann potential. Also, when $b = c = \delta = 0$, Eq. (1) reduces to Coulomb potential.

Fig. 1. Variation of the class of Yukawa potential as a function of r .

The aim of this work is to investigate the SE with the class of Yukawa potential model in the framework of series expansion method to obtain the mass spectra of quark-antiquark systems. To the best of our knowledge this is the first time class of Yukawa potential model is being studied with the aim of determining the mass spectra of heavy quarkonia systems.

The paper is organized as follows: In section 2, the bound state energy eigenvalues is calculated via series expansion method. In section 3, the results are discussed. In section 4, the conclusion is presented.

II. APPROXIMATE SOLUTIONS OF THE SCHRÖDINGER EQUATION WITH CLASS OF YUKAWA POTENTIAL MODEL USING SERIES EXPANSION METHOD

We consider the radial SE of the form [28],

$$\frac{d^2 R(r)}{dr^2} + \frac{2}{r} \frac{dR(r)}{dr} + \left[\frac{2\mu}{\hbar^2} (E_{nl} - V(r)) - \frac{l(l+1)}{r^2} \right] R(r) = 0 \quad (2)$$

where l is angular quantum number taking the values $0, 1, 2, 3, 4, \dots$, μ is the reduced mass for the quarkonium

particle, r is the inter-nuclear separation and E_{nl} denotes the energy eigenvalues of the system.

In order to make Eq. (1) temperature-dependent, the screening parameter is replaced with Debye mass ($m_D(T)$) which is temperature-dependent and vanishes at $T \rightarrow 0$, this gives:

$$V(r, T) = -\frac{a}{r} + \frac{be^{-m_D(T)r}}{r} - \frac{ce^{-2m_D(T)r}}{r^2} \quad (3)$$

We carry out Taylor series expansion of the exponential terms in Eq. (3) up to order three, in order to make the potential to interact in the quark-antiquark system and this yield:

$$\frac{e^{-m_D(T)r}}{r} = \frac{1}{r} - m_D(T) + \frac{m_D^2(T)r}{2} - \frac{m_D^3(T)r^2}{6} + \dots \quad (4)$$

$$\frac{e^{-2m_D(T)r}}{r^2} = \frac{1}{r^2} - \frac{2m_D(T)}{r} + 2m_D^2(T) - 1.33m_D^3(T)r \quad (5)$$

By substituting Eqs. (4) and (5) into Eq. (3) we have:

$$V(r, T) = -\frac{\alpha_0}{r} + \alpha_1 r + \alpha_2 r^2 + \frac{\alpha_3}{r^2} + \alpha_4 \quad (6)$$

where

$$\left. \begin{aligned} -\alpha_0 &= a - b - 2cm_D(T), \quad \alpha_1 = \frac{bm_D^2(T)}{2} - 1.33m_D^3(T) \\ \alpha_2 &= -\frac{bm_D^3(T)}{6}, \quad \alpha_3 = -c, \quad \alpha_4 = -bm_D(T) - 2cm_D^2(T) \end{aligned} \right\} \quad (7)$$

We substitute Eq. (6) into Eq. (2) and obtain:

$$\frac{d^2 R(r)}{dr^2} + \frac{2}{r} \frac{dR(r)}{dr} + \left[\varepsilon + \frac{A}{r} - Br - Cr^2 - \frac{L(L+1)}{r^2} \right] R(r) = 0 \quad (8)$$

where

$$\left. \begin{aligned} \varepsilon &= \frac{2\mu}{\hbar^2} (E - \alpha_4), \quad A = \frac{2\mu\alpha_0}{\hbar^2} \\ B &= \frac{2\mu\alpha_1}{\hbar^2}, \quad C = \frac{2\mu\alpha_2}{\hbar^2} \end{aligned} \right\} \quad (9)$$

$$L(L+1) = \frac{2\mu\alpha_3}{\hbar^2} + l(l+1) \quad (10)$$

From Eq. (10), we have:

$$L = -\frac{1}{2} + \frac{1}{2} \sqrt{(2l+1)^2 + \frac{8\mu\alpha_3}{\hbar^2}} \quad (11)$$

Now make an anzats wave function [38],

$$R(r) = e^{-\alpha r^2 - \beta r} F(r) \quad (12)$$

where α and β are positive constants whose values are to be determined in terms of potential parameters. Differentiating Eq. (12), we obtain:

$$R'(r) = F'(r)e^{-\alpha r^2 - \beta r} + F(r)(-2\alpha r - \beta)e^{-\alpha r^2 - \beta r} \quad (13)$$

$$R''(r) = F''(r)e^{-\alpha r^2 - \beta r} + F'(r)(-2\alpha r - \beta)e^{-\alpha r^2 - \beta r} + [(-2\alpha) + (-2\alpha r - \beta)(-2\alpha r - \beta)]F(r)e^{-\alpha r^2 - \beta r} \quad (14)$$

By substituting Eqs. (12), (13) and (14) into Eq. (8) and dividing through by $e^{-\alpha r^2 - \beta r}$ we obtain:

$$F''(r) + \left[-4\alpha r - 2\beta + \frac{2}{r} \right] F'(r) + \left[\frac{(4\alpha^2 - C)r^2 + (4\alpha\beta - B)r}{r} + (A - 2\beta)\frac{1}{r} - \frac{L(L+1)}{r^2} + (\varepsilon + \beta^2 - 6\alpha) \right] F(r) = 0 \quad (15)$$

$$\sum_{n=0}^{\infty} (2n+L)(2n+L-1)a_n r^{2n+L-2} + \left[-4\alpha r - 2\beta + \frac{2}{r} \right] \sum_{n=0}^{\infty} (2n+L)a_n r^{2n+L-1} + \left[(4\alpha^2 - C)r^2 + (4\alpha\beta - B)r + \frac{(A-2\beta)}{r} - \frac{L(L+1)}{r^2} + (\varepsilon + \beta^2 - 6\alpha) \right] \sum_{n=0}^{\infty} a_n r^{2n+L} = 0 \quad (19)$$

By collecting powers of r in Eq. (19) we have:

$$\sum_{n=0}^{\infty} a_n \left\{ \left[(2n+L)(2n+L-1) + 2(2n+L) - L(L+1) \right] r^{2n+L-2} + \left[-2\beta(2n+L) + (A-2\beta) \right] r^{2n+L-1} + \left[-4\alpha(2n+L) + \varepsilon + \beta^2 - 6\alpha \right] r^{2n+L} + [4\alpha\beta - B] r^{2n+L+1} + [4\alpha^2 - C] r^{2n+L+2} \right\} = 0 \quad (20)$$

Equation (20) is linearly independent implying that each of the terms is separately equal to Zero, noting that r is a non-zero function; therefore, it is the coefficient of r that is zero. With this in mind, we obtain the relation for each of the terms.

$$(2n+L)(2n+L-1) + 2(2n+L) - L(L+1) = 0 \quad (21)$$

$$-2\beta(2n+L) + A - 2\beta = 0 \quad (22)$$

$$-4\alpha(2n+L) + \varepsilon + \beta^2 - 6\alpha = 0 \quad (23)$$

$$4\alpha\beta - B = 0 \quad (24)$$

$$4\alpha^2 - C = 0 \quad (25)$$

From Eqs. (22) and (25) we have:

$$\beta = \frac{A}{4n+2L+2} \quad (26)$$

$$\alpha = \frac{\sqrt{C}}{2} \quad (27)$$

We proceed to obtaining the energy eigenvalues equation using Eq. (23) and have:

$$\varepsilon = 2\alpha(4n+2L+3) - \beta^2 \quad (28)$$

The function $F(r)$ is considered as a series of the form:

$$F(r) = \sum_{n=0}^{\infty} a_n r^{2n+L} \quad (16)$$

Taking the first and second derivatives of Eq. (16) we obtain:

$$F'(r) = \sum_{n=0}^{\infty} (2n+L)a_n r^{2n+L-1} \quad (17)$$

$$F''(r) = \sum_{n=0}^{\infty} (2n+L)(2n+L-1)a_n r^{2n+L-2} \quad (18)$$

We substitute Eqs. (16), (17) and (18) into Eq. (15) and obtain:

$$F''(r) + \left[-4\alpha r - 2\beta + \frac{2}{r} \right] F'(r) + \left[\frac{(4\alpha^2 - C)r^2 + (4\alpha\beta - B)r}{r} + (A - 2\beta)\frac{1}{r} - \frac{L(L+1)}{r^2} + (\varepsilon + \beta^2 - 6\alpha) \right] F(r) = 0 \quad (19)$$

$$F''(r) + \left[-4\alpha r - 2\beta + \frac{2}{r} \right] F'(r) + \left[\frac{(4\alpha^2 - C)r^2 + (4\alpha\beta - B)r}{r} + (A - 2\beta)\frac{1}{r} - \frac{L(L+1)}{r^2} + (\varepsilon + \beta^2 - 6\alpha) \right] F(r) = 0 \quad (20)$$

By substituting Eqs. (9), (11), (26) and (27) into Eq. (28) and simplifying we obtain:

$$E_{nl} = \sqrt{\frac{\hbar^2 \alpha_2}{2\mu}} \left(4n+2 + \sqrt{(2l+1)^2 + \frac{8\mu\alpha_3}{\hbar^2}} \right) - \frac{2\mu\alpha_0^2}{\hbar^2} \left(4n+1 + \sqrt{(2l+1)^2 + \frac{8\mu\alpha_3}{\hbar^2}} \right)^{-2} + \alpha_4 \quad (29)$$

Upon substituting Eq. (7) into Eq. (29) we obtain the energy eigenvalues of the class of Yukawa potential as:

$$E_{nl} = \sqrt{\frac{-\hbar^2 b m_D^3(T)}{12\mu}} \left(4n+2 + \sqrt{(2l+1)^2 - \frac{8\mu c}{\hbar^2}} \right) - \frac{2\mu}{\hbar^2} (b-a+2cm_D(T))^2 \left(4n+1 + \sqrt{(2l+1)^2 - \frac{8\mu c}{\hbar^2}} \right)^{-2} - bm_D(T) - 2cm_D^2(T) \quad (30)$$

III. SPECIAL CASE

When we set $c = 0$, we obtain the energy eigenvalues for Hellmann potential:

$$E_{nl} = \sqrt{\frac{-\hbar^2 b m_D^3(T)}{12\mu}} \left(4n+2 + \sqrt{(2l+1)^2} \right) - \frac{2\mu}{\hbar^2} (b-a)^2 \left(4n+1 + \sqrt{(2l+1)^2} \right)^{-2} - bm_D(T) \quad (31)$$

When we set $c = b = 0$, we obtain the energy eigenvalues for Coulomb potential:

$$E_{nl} = -\frac{2\mu a^2}{\hbar^2} \left(4n+1 + \sqrt{(2l+1)^2} \right)^{-2} \quad (32)$$

IV. RESULTS AND DISCUSSION

A. Results

The mass spectra of the heavy quarkonium system such as charmonium and bottomonium that have the quark and antiquark flavor is calculated and we apply the following relation [39], [40]:

$$M = 2m + E_{nl} \quad (33)$$

Where m is quarkonium bare mass, and E_{nl} is energy eigenvalues. By substituting Eq. (30) into Eq. (33) we obtain the mass spectra for Varshni potential as:

$$M = 2m + \left\{ \frac{-\hbar^2 b m_D^3(T)}{12\mu} \left(4n+2 + \sqrt{(2l+1)^2 - \frac{8\mu c}{\hbar^2}} \right) - \frac{2\mu}{\hbar^2} (b-a+2cm_D(T))^2 \right. \\ \left. \left(4n+1 + \sqrt{(2l+1)^2 - \frac{8\mu c}{\hbar^2}} \right)^{-2} - b m_D(T) - 2cm_D^2(T) \right\} \quad (34)$$

TABLE 1: MASS SPECTRA OF CHARMONIUM IN (GeV) ($m_c = 1.209$ GeV, $\mu = 0.6045$ GeV, $a = 0.489$ GeV, $b = -0.695$ GeV, $c = 5.679$ GeV, $m_D(T) = 1.52$ GeV, $\hbar = 1$)

State	Present work	NU [40]	[30] AIM	Experiment [42]
1S	3.096	3.096	3.096	3.096
2S	3.686	3.686	3.672	3.686
1P	3.524	3.255	3.521	3.525
2P	3.773	3.779	3.951	3.773
3S	4.040	4.040	4.085	4.040
4S	4.263	4.269	4.433	4.263
1D	3.769	3.504	3.800	3.770
2D	4.156	-	-	4.159
1F	4.081	-	-	-

TABLE 2: MASS SPECTRA OF BOTTOMONIUM IN (GeV), ($m_b = 4.823$ GeV, $\mu = 2.4115$ GeV, $a = 1.192$ GeV, $b = 0.998$ GeV, $c = 3.876$ GeV, $m_D(T) = 1.52$ GeV, $\hbar = 1$)

State	Present work	[40] NU	[30] AIM	Experiment [42]
1S	9.460	9.460	9.462	9.460
2S	10.023	10.023	10.027	10.023
1P	9.889	9.619	9.963	9.899
2P	10.260	10.114	10.299	10.260
3S	10.355	10.355	10.361	10.355
4S	10.579	10.567	10.624	10.580
1D	10.164	9.864	10.209	10.164
2D	10.575	-	-	-
1F	10.299	-	-	-

Fig. 2. Variation of mass spectra with the reduced mass (μ) for different quantum numbers.

Fig. 3. Variation of mass spectra with potential strength (a) for different quantum numbers.

B. Discussion

We calculate mass spectra of charmonium and bottomonium for states from 1S to 1F, by using Eq. (34). The free parameters of Eq.(34) were then obtained by solving two algebraic equations by inserting experimental data of mass spectra for 2S, 2P in Eq.(34) in the case of charmonium. In the case of bottomonium the values of the free parameters are calculated by solving two algebraic equations, which were obtained by inserting experimental data of mass spectra for 1S, 2S.

For bottomonium $b\bar{b}$ and charmonium $c\bar{c}$ systems we adopt the numerical values of these masses as $m_b = 4.823$ GeV and $m_c = 1.209$ GeV respectively [41]. Then, the corresponding reduced mass are $\mu_b = 2.4115$ GeV and $\mu_c = 0.6045$ GeV. The Debye mass $m_D(T)$ is taken as 1.52 GeV by fitted with experimental data. The experimental data were taken from [42]. We note that calculation of mass spectra of charmonium and bottomonium are in good agreement with experimental data, as shown in Tables 1 and 2. The values obtained are also in good agreement with work of other researchers like; [40] as shown in Tables 1 and 2 in which the author investigated the N- radial SE analytically. The Cornell potential was extended to finite temperature.

We also plotted mass spectra energy against potential strength (a), and reduced mass (μ), respectively. In Fig. 2,

the mass spectra energy converges at the beginning and later spreads out, and there is a monotonic increase in potential strength (a) and the quantum number increases. Fig. 3 shows the convergence of the mass spectra energy as the reduced mass (μ) increases for various angular quantum numbers.

V. CONCLUSION

In this study, we adopted a class of Yukawa potential for quark-antiquark interactions. The potential model was made to be temperature-dependent by replacing the screening parameter with Debye mass. We obtained the approximate solutions of the Schrödinger equation for energy eigenvalues using the series expansion method. Two special cases were considered which result to: Hellmann and Coulomb potentials. We applied the present results to compute heavy-meson masses of charmonium and bottomonium for different quantum states. The result agreed with experimental data and work of other researchers. Also, we have discussed the plots obtained graphically. The exponential type potential has been successfully applied in predicting the mass spectra of heavy mesons. The analytical solutions can also be used to describe other characteristics of the quarkonium systems like thermodynamic properties.

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