European Journal of Applied Physics
ISSN: 2684-4451

A Short Review on the Preferential Concentration of Particles in Fluid Flow

Md. Shahraj Chowdhury

***Abstract* — Particle laden fluid flows are important in many fields of application such as environmental, chemical, astrophysical, and biological flows. They are examples of multicomponent fluid flow where the dispersed component is transported within a carrier fluid, and the dynamics of the flow are mostly dictated by the carrier fluid. The dispersed phase generally consists of particles such as droplets, bubbles, sediments, or nanoparticles while the carrier phase is typically water, oil, and chemical and biological liquids. Preferential concentration is an important characteristic of such multicomponent fluid flow which is defined by a higher density of particles in local regions of flow based on local acceleration, vorticity, strain, and pressure. Due to the intrinsic challenges of dealing with turbulence, mixing, particle tracking, and inter-and intra-component interactions, these flows are complicated to model. Based on the mass ratio of the particles to the fluid, the particles exhibit different preferential concentration patterns. In this article, experimental and numerical works on the preferential concentration of dispersed particles in different fluid flow problems are reviewed and discussed.**

Keywords — Fluid Dynamics, Particles, Preferential Concentration, Turbulent Flow.

I. INTRODUCTION

Multicomponent fluid flow is characterized by flow within a carrier fluid such as water with one or more dispersed particle phases. Investigation of such particle transport by fluids offers not only an understanding of different natural systems, such as sediment transport in rivers and dust transport in atmospheres, but also man made artificial engineering systems, such as the production of chemical suspensions and clean room filtration devices. It is considered one of the most classical problems in fluid dynamics, especially in turbulence research [1]. However, the thorough dynamics of this phenomenon are yet to be understood and numerical modeling of such transport in full-scale simulations is almost impossible to perform with the current state of the art of scientific computation.

Preferential concentration is an important feature of multicomponent fluid flow systems. It is the tendency of particles in a fluid to cluster in regions based on their mass, size, and local fluid properties such as strain, vorticity, acceleration, and pressure. In laminar flow, such concentration is dictated by the effect of gravity on the particles. In turbulent flow, the importance of gravity is lesser compared to the local flow dynamics. Stokes number is often used to determine the extent of such clustering, particles with $St \ll 1$ tend to follow fluid streamlines where particles with $St \gg 1$ do not significantly respond to the fluid within the times the fluid motions are coherent [2]. The dynamics of preferential concentration can significantly influence different fluid dynamics systems, for example, cloud droplet formation [2], respiratory droplet transport [2], aerosol and suspension production and transport [2], cavitation [3], [4], and even formation of planet and nebulae [2].

Experimental studies have been carried out to study the preferential concentration of heavy, neutral, and light particles in fluids. Similar studies have been performed numerically at different Reynolds numbers. Since particle concentration is mostly associated with turbulent flow, only studies of preferential concentration in turbulent flow will be discussed in this article. Experimental investigations will be reviewed at first and then numerical efforts will be discussed.

II. EXPERIMENTAL STUDIES

To comprehend the processes of preferred concentrations in turbulent flow, several experiments have been carried out. Early research concentrated on the region relatively close to the wall, where particle concentrations in low-speed streaks had the primary influence. Young & Hanratty [5] and Rashidi *et al.* [6] looked at the concentration field in water flows that are wall-bounded. According to Young and Hanratty's experiment, particles from a fluid that was turbulently flowing downward were caught in a fluid that was

---

Submitted on June 22, 2022.
Published on July 20, 2022.
M. S. Chowdhury, Khulna University of Engineering & Technology, Khulna, Bangladesh.
(corresponding e-mail: shahrajch@gmail.com).

moving slowly close to the wall. One particle diameter separated us from the wall. The concentrated particles form patterns that resemble necklaces and have a spanwise spacing akin to low-speed streaks near a wall. The Saffman lift force, which the authors believed to be the origin of this incident, created a drift velocity towards the direction of the wall. They came to the conclusion that the Saffman lift force balances the fluid displacement force at the particle's distance from the wall.

In free shear layers, experimental investigations of particle diffusion have correlated the particle concentration field to the large-scale vertical structures [7]. For instance, Lazaro and Lasheras [8] investigated the causes of particle dispersion in an inhomogeneous, anisotropic, turbulent, high Reynolds number free shear flow. They discovered that a Kelvin-Helmholtz instability of the sort that causes a selective dispersion of the particles across the mixing layer is what causes the spectrum response of the particles to the coherent velocity field that emerges in the layer. This dispersion layer is made up of two peripheral sublayers with noticeably bigger mean particle sizes and a central core area with modest mean particle size. According to their experimental findings, coherent, massive vortical structures predominate in particle dispersion. But on the other hand, Longmire and Eaton's experimental investigation of particle dispersion in a round jet has shown that dense particles tend to cluster in the saddle areas in between succeeding vortex rings [9].

In a turbulent channel flow, Fessler *et al*. [10] studied heavy particles and their preferential behavior. To acquire the concentration field, they took pictures of particles irradiated by a spanwise laser sheet. Compared to random distribution, significant departures have been found and they attributed this difference to the time scales of the particles. The maximum preferred concentration was observed at a ratio of around one ($St{\sim}1$) between the particle's aerodynamic reaction time and the flow's Kolmogorov time scale. Additionally, the length scales of the particle clusters were studied and were shown to vary with particle size. The inter-cluster spacing was also relatively larger than the scales of preferential concentration. A similar set of experiments in microgravity conditions has been performed by Fallon & Rogers [11]. They investigated how gravity affects preferred concentration and its mechanisms. Their research supported Fessler *et al*.'s conclusion that preferable concentration depends on the Stokes number.

Inertial particle clustering in nearly isotropic turbulence was compared in depth between direct numerical modeling and tests in Salazar *et al*.'s work [12]. Three-dimensional digital holographic particle imaging was utilized to map the positions of several hundred particles in a volume of 1 cm^3 in the middle of the box using hollow glass spheres with a mean diameter of 6 μm. In their experiments, the authors find strong agreement between the DNS observations and the radial distribution function (RDF) measure.

Aliseda *et al*. conducted an experimental study on the behavior of heavy particles in isotropic, homogenous, decaying turbulence [13]. They noticed that the particle settling velocity is substantially higher than in a quiescent fluid. They concluded that settling velocity depends on particle loading and rises as particle volume fraction rises. By examining the spatial and temporal distribution of the particle concentration field, significant inhomogeneities have also been discovered. The distinctive dimension of preferentially concentrated particle clusters in relation to the viscosity scales of the flow was identified by the authors. By measuring the settling velocity conditional on the local concentration of particles in the flow, a monotonic rise in the settling velocity with the local concentration has been found. The behavior of tiny spherical bubbles submerged in a homogenous isotropic turbulent carrier flow of a heavier fluid was further investigated by Aliseda and Lasheras [14] in order to conduct an experimental investigation on the decrease in preferred concentration and rising velocity. They introduced air bubbles with sizes ranging from 10 to 900 microns into the test area of a horizontal water channel and then monitored how the turbulence interacted with the bubbles. A grid that was placed at the test section's entrance created turbulence. Using data from light interferometry and flow visualizations that displayed the bubbles' instantaneous concentration field in the carrier flow, measurements of the bubble diameter, convective and rising velocities, and bubble diameter was made. It was discovered that the turbulence's impact on the bubbles altered their concentration field, causing preferred accumulation at tiny scales. The rise velocity of bubbles in the flow was reduced by preferential concentration below the value seen and expected for bubbles in a stationary fluid. The authors also explained the results in terms of forces acting on the bubble in an inhomogeneous flow and the outcome of pressure fluctuations that preferentially drive bubbles to the center of vortices.

Experimental research was conducted by Qureshi *et al*. [15] on the Lagrangian statistics of neutrally buoyant, finite-sized particles moving through an isotropic turbulent flow. By changing the particle diameter over turbulent inertial scales, they investigated the impact of limited size. They discovered that velocity intermittency is not trivially connected to finite-size effects. Additionally, while the particle's acceleration variance reduces with increasing size, the global form of the particle's acceleration probability density functions does not greatly rely on its size. For conditions of continuous fluid flow, the acceleration statistics of inertial particles of finite size were particularly resistant to fluctuations in size and density, the impact of which is mostly conveyed by acceleration variance alone. Over the whole range of densities investigated, the form of the acceleration probability density function does not vary, much like the impact of increasing size [16]. The authors came to the conclusion that neither the heavy point particle limit nor the finite-size neutrally buoyant case can trivially extrapolate finite size effects for heavy particles because both of these limitations foretell a monotonic drop in acceleration variance with increasing inertia. This runs counter to measured trends for heavy particles of limited size. As a result, unlike in the case of point particles, finite size and density effects cannot be explained by straightforward filtering arguments based merely on Stokes number effects.

Guala *et al.* [17] have undertaken experimental research on the clustering of large particles in homogeneous isotropic flow. At the Taylor microscale Reynolds number $Re_\lambda = 250$, they presented the findings of simultaneous measurements of fluid flow and motion of big solid particles in a homogeneous turbulent flow. They measured the velocity, acceleration, and spatial distribution of particles in a three-dimensional volume as well as the surrounding flow velocity and velocity derivative fields like vorticity and the strain-rate tensor using two synchronized and cross-calibrated three-dimensional particle tracking velocimetry (3D-PTV) systems. They concentrated on the two-way coupling between the turbulent flow and solid particles that were heavier than the surrounding liquid ($\rho_p = 1400$ kg m^{-3}, $\rho_f = 1000$ kg m^{-3}) and substantially larger (~900 μm) than the Kolmogorov length scale (~200 μm). Large particles tend to cluster in locations where strain is dominant, and preferential concentration occurs on scales equivalent to the Taylor microscale (λ), according to the combined statistics of the local particle concentration, the local strain, and the local vorticity fields. Finally, they concluded that the observed clustering of large particles may well be connected to the same dynamics that apply to the clustering of sub-Kolmogorov size particles and that the time scale $\lambda/u_{r.m.s.}$ can substitute the Kolmogorov time scale as a normalization of the particle response time in the determination of the Stokes number.

The preferential concentration of poly-dispersed water droplets has been studied experimentally by Lian *et al* [18] in stationary homogeneous isotropic turbulence at turbulent Reynolds numbers of $Re_\lambda = 107$, 145, 185, and 213. The droplets studied had Sauter mean diameters ranging from 25 to 95 μm. To assess the amount of preferred droplet concentration, the radial distribution function (RDF) and 2D Voronoi analysis were applied, and the findings from both techniques were in good agreement. According to their RDF results, over all experimental settings, the typical length scale of resultant droplet clusters varied between 20 and 30 times the Kolmogorov length scale. Furthermore, it was shown that preferred concentration is more closely connected to Stokes number. For all experimental settings, the size of droplet preferred concentration was greatest when the Stokes number was close to unity. The magnitude of preferential concentration was not very related to turbulent Reynolds numbers, contrary to most of the literature. Sumbekova *et al* [19] found out that for the polydisperse size distributions, clustering is strongly enhanced (quasi-linearly) by Re_λ and noticeably enhanced (with a square-root dependence) with void fraction. The cluster and void sizes are also scaled with the Kolmogorov length scale η and are dictated primarily by Re_λ. In this study, the range of Reynolds number was between 170 and 450, while Stokes number and void fraction ranged between 0.1-5 and 2×10^{-6} and 2×10^{-5}, respectively.

Regarding the experimental techniques used till date, Toschi & Bodenschatz [20] and Brandt & Coletti [21] provided a comprehensive review of the literature. Readers are encouraged to go through their articles for more information.

III. NUMERICAL STUDIES

Numerical studies based on DNS and LES to study preferential concentration of particles in turbulent flow are at large in the literature. Numerical investigations of the particle concentration in turbulence solve the Navier-Stokes equation and use the Eulerian velocity field u(x, t) to evolve the trajectories of tracers x(t), integrating the transport equation. For particles with inertia, drag and other force terms such as gravity appears in the computation [4]. The evolution of large-scale supercomputers in combination with efficient algorithms has enabled numerical simulations of turbulent flow and transport of millions of particles. Most commonly used techniques include standard pseudo-spectral methods for the Eulerian field with dealiasing [22]. Most of the studies are DNS of turbulent flow coupled with point-particle transport; they are suitable for particles smaller than the smallest flow scales. However, for larger particles, particle-resolved DNS are more preferred. The study of the preferential concentration of particles in combination with DNS of turbulent flow started a few decades ago. Early works such as Maxey and Riley [23], Maxey [24], Squires & Eaton [7], Ruetsch & Meiburg [25], and Wang & Maxey [26] are the pioneers in this field of study.

The mean settling velocity in homogeneous turbulence of a tiny rigid spherical particle exposed to a Stokes drag force differs from that in still fluid due to a bias in particle inertia [24]. Squires and Eaton explored the influence of turbulence on particle concentration fields as well as turbulence modification utilizing DNS of isotropic turbulence [27]. The particle velocity was computed using Stokes' drag law, and the volume fraction of particles was considered to be insignificant. In simulations when the particles have no effect on turbulence, they discovered that light particles concentrate preferentially in regions with low vorticity and high strain rate. The particles attenuated a greater proportion of the turbulent energy as the mass loading increased. Furthermore, light particles modified turbulence differently than heavy particles because light particles gathered in low-vorticity, high-strain-rate zones. Squires and Eaton used DNS of isotropic turbulence to evaluate the influence of turbulence on the concentration fields of heavy particles in another work [7]. The particle motion in the simulations was modeled using the Stokes drag equation and was assumed to be substantially denser than the fluid. One million particles were seeded into the flow, and their paths were tracked over the simulated flow fields. To calculate the large-scale turbulence time scale, the authors employed three particle time constants: 0.075, 0.15, and 0.52. Their simulations confirmed that particles aggregate preferentially in low vorticity and high strain rate locations. The intermediate particle time constant showed the highest preferred concentration (0.15). The particles' instantaneous number density reached 25 times the usual value. They determined that turbulence may impede rather than increase particle mixing since they discovered dense particles to cluster in places of low vorticity and high strain.

The more intense vorticity in a turbulent flow, according to Ruetsch and Maxey [28], has a tendency to form coherent, confined, and tube-like vortical structures at dissipation-range scales. As a result, small-scale flow dynamics contribute significantly to particle accumulation at the local level, and it is most likely connected to particle-flow vorticity interactions. Later, Ruetsch and Meiburg [25] used numerical simulations to investigate the dynamics of small, spherical noninteracting bubbles in two-dimensional vortical flows. They calculated bubble trajectories in a solid-body vortex and discovered that bubble motion is linked to the location of bubble accumulation and the rate of accumulation. They proposed that for some value of the inertia parameter, or inverse Stokes number, the rate of entrapment into the vortex has an optimum. They also studied bubble motion in a temporally developing shear layer, where their solid-body vortex model anticipated trends in increasing concentration around the vortex center in the absence of gravity. In the absence of gravity, some bubbles escape the vortex, and the fraction of caught bubbles increases as the inertia value decreases.

Using DNS of homogeneous isotropic turbulence and heavy particles, Wang and Maxey showed that a major increase in the mean settling velocity can occur for particles with inertial response time and still-fluid terminal velocity comparable to the Kolmogorov scales of the turbulence. This increase was found to be as high as 50% of the terminal velocity.

Large Eddy Simulation (LES) was used by Wang and Squires [29] to model fully-developed turbulent channel flow and particle transport. Based on friction velocity and channel half width, the corresponding Reynolds numbers, Re_τ, were 180 and 644. The Lagrangian dynamic eddy viscosity model was used to calculate their subgrid-scale stresses. Drag and gravitational forces regulated particle motion, and the volume fraction of the dispersed phase was insignificant in terms of carrier flow parameters. Dispersed phase statistical features agreed well with the DNS at Re_τ=180, while acceptable agreement was established between the LES and experimental observations at Re_τ=644. The analysis of preferred particle concentration by turbulence near the wall and along the channel centerline is qualitatively and quantitatively similar to that reported in DNS and the experiments of Fessler *et al*. [10].

Based on the friction velocity and the channel half-width, Rouson and Eaton [30] performed DNS of the passive transport of solid particles by a fully developed turbulent channel flow with a Reynolds number of 180. They looked at three particle sizes with viscous wall units ranging from 0.5 to 1.4 and aerodynamic time constants between 0.6 and 56 centerline Kolmogorov time scales. They have demonstrated that the particle spatial concentration peaks close to a Stokes number of unity using particle number density histograms and fractal dimensions. Their findings suggest that, in the presence of streamwise gravitational acceleration, the preferential concentration of particles in low-speed streaks causes suppression of particle velocities in the viscous sublayer and buffer zone. In other areas of the flow, the non-random particle distribution was unrelated to the local flow topology. Results were in accordance with Fessler *et al*.'s experimental data [10].

For the study of one-way coupling DNS data of heavy particles settling in homogeneous turbulence, Dejoan and Monchaux [31] employed Voronoi diagrams. To investigate the preferential concentration and clustering of the inertial particles, a wide variety of particle Stokes and Rouse numbers were taken into consideration. To investigate the impact of preferred concentration on the acceleration of settling velocity, the authors employed statistics of particle and flow field quantities conditional on the local concentration. However, their conditional statistics indicated that the gravitational effects had resulted in a fine-tuned preferential sampling of the flow field. They did not detect any appreciable gravitational impacts on the global features of preferential concentration. This result suggests that both the falling fluid velocity and acceleration have an impact on the increased settling velocity. They discovered that the clusters may be modeled as 2D elongated manifolds. In both zero and non-zero gravity fields, they had a similar form, but for Stokes numbers close to unity, the Voronoi cells grew longer. Additionally, the gravitational forces favor particle orientation perpendicular to the gravitational axis.

Wang *et al*. [32] studied inertial particles of two different volume fractions (3×10^{-6} and 5×10^{-5}) in a turbulent vertical channel flow at a Stokes number of 58.6. They compared two independent DNS models with the point-particle approach to experimental measurements. For low volume fraction, both models and experimental data agree well, however, near the walls for the wall-normal particle velocity fluctuations did not match very well. These differences increased for the high volume fraction and particle concentration is overpredicted in DNS.

Bappy *et al*. conducted several numerical studies on the Lagrangian transport of tracers and finite sized bubbles in homogeneous isotropic turbulence [33]-[35]. Comparing Re_λ of 150 to 418, they have found that for tracer particles higher concentrations are observed at higher Taylor microscale Reynolds numbers [4]. Their subsequent studies were concerned with preferential concentrations of finite-sized bubbles in low-pressure regions of flow concerned with cavitation inception. Transporting 64^3 particles in homogeneous isotropic turbulence at $Re_\lambda = 150$, they investigated the effects of gravity and bubble size on the preferential concentrations in the low-pressure vortical regions. Their results show that the concentration rate increases with increasing bubble size while it decreases with increasing gravity parameter [34]. Finally, they have presented similar results of higher concentration of bubbles in low-pressure regions of turbulent flow at a range of Reynolds numbers from 22 up to 240 [35].

For a more comprehensive review, the readers are referred to Balachandar & Eaton [36] and Brandt & Coletti [21]. They have extensively discussed the current state of the art and potential challenges to overcome in this field.

IV. Discussion and Conclusion

Preferential concentration is a characteristic feature of multicomponent turbulent fluid flow which is defined by a higher concentration of particles in local regions of flow based on acceleration, vorticity, strain, and pressure. Numerous numerical and experimental works have been carried out to better understand preferential concentration, what flow features affect them the most, and if the particles modify the fluid flow. Two general properties of such concentrations are: the rate of preferential concentration is highest at a Stokes number of 1 and the rate increases as the Reynolds number of the flow increases. These particles behave differently based on their density relative to the carrier fluid. Heavier particles concentrate away from the core of turbulent eddies in regions of low vorticity of the carrier flow due to centrifugal expulsion. On the other hand, lighter particles tend to gather in high vorticity regions near the vortex cores due to centripetal force and pressure gradient. Also, gravity plays an important role in this clustering; higher gravitational pull reduces particle concentrations in the vortex cores (for light particles) and in the high strain regions (for heavy particles). The study of preferential concentration is limited to moderate Reynolds number so far. With the advancement of computational power, we need to gradually increase this level and have insights into the behavior of particles at higher turbulence and with a larger range of particles. With the proper study of this phenomenon, engineers and scientists will be able to invent new methods and applications concerning different natural, biological, hydraulic, naval, and other engineering systems.

Funding

The author did not receive any financial support from any funding agencies.

Conflict of Interest

Author declares no conflict of interest.

References

[1] Rouson DW, Eaton JK. On the preferential concentration of solid particles in turbulent channel flow. *Journal of Fluid Mechanics*. 2001;428:149-69.

[2] Wikipedia contributors. Preferential concentration [Internet]. Wikipedia, The Free Encyclopedia. 2022. Available from: https://en.wikipedia.org/w/index.php?title=Preferential_concentration&oldid=1065197160.

[3] Bappy M, Martin JE, Li J, Buscaglia GC, Carrica PM. A stochastic approach to cavitation inception prediction. *InProceedings of the Sixth International Symposium on Marine Propulsors,* 2019:91-98.

[4] Bappy M, Carrica PM, Buscaglia GC. Lagrangian statistics of pressure fluctuation events in homogeneous isotropic turbulence. *Physics of Fluids*. 2019;31(8):085111.

[5] Young JB, Hanratty TJ. Optical studies on the turbulent motion of solid particles in a pipe flow. *Journal of Fluid Mechanics*. 1991;231:665-88.

[6] Rashidi M, Hetsroni G, Banerjee S. Particle-turbulence interaction in a boundary layer. *International Journal of Multiphase Flow*. 1990;16(6):935-49.

[7] Squires KD, Eaton JK. Preferential concentration of particles by turbulence. *Physics of Fluids A: Fluid Dynamics*. 1991;3(5):1169-78.

[8] Lazaro BJ, Lasheras JC. Particle dispersion in a turbulent, plane, free shear layer. *Physics of Fluids A: Fluid Dynamics*. 1989;1(6):1035-44.

[9] Longmire EK, Eaton JK. *Proceedings of the Korea-U.S. Fluids Engineering Seminar*, edited by J. Kim (Korea Institute of Technology, Taejon, Korea, 1989), pp. 435–454.

[10] Fessler JR, Kulick JD, Eaton JK. Preferential concentration of heavy particles in a turbulent channel flow. *Physics of Fluids*. 1994;6(11):3742-9.

[11] Fallon T, Rogers CB. Turbulence-induced preferential concentration of solid particles in microgravity conditions. *Experiments in fluids*. 2002;33(2):233-41.

[12] Salazar JP, De Jong J, Cao L, Woodward SH, Meng H, Collins LR. Experimental and numerical investigation of inertial particle clustering in isotropic turbulence. *Journal of Fluid Mechanics*. 2008;600:245-56.

[13] Aliseda A, Cartellier A, Hainaux F, Lasheras JC. Effect of preferential concentration on the settling velocity of heavy particles in homogeneous isotropic turbulence. *Journal of Fluid Mechanics*. 2002;468:77-105.

[14] Aliseda A, Lasheras JC. Preferential concentration and rise velocity reduction of bubbles immersed in a homogeneous and isotropic turbulent flow. *Physics of Fluids*. 2011 6;23(9):093301.

[15] Qureshi NM, Bourgoin M, Baudet C, Cartellier A, Gagne Y. Turbulent transport of material particles: an experimental study of finite size effects. *Physical review letters*. 2007;99(18):184502.

[16] Qureshi NM, Arrieta U, Baudet C, Cartellier A, Gagne Y, Bourgoin M. Acceleration statistics of inertial particles in turbulent flow. *The European Physical Journal B*. 2008;66(4):531-6.

[17] Guala M, Liberzon A, Hoyer K, Tsinober A, Kinzelbach W. Experimental study on clustering of large particles in homogeneous turbulent flow. *Journal of Turbulence*. 2008;1(9):N34.

[18] Lian H, Charalampous G, Hardalupas Y. Preferential concentration of poly-dispersed droplets in stationary isotropic turbulence. *Experiments in fluids*. 2013;54(5):1-7.

[19] Sumbekova S, Cartellier A, Aliseda A, Bourgoin M. Preferential concentration of inertial sub-Kolmogorov particles: the roles of mass loading of particles, Stokes numbers, and Reynolds numbers. *Physical Review Fluids*. 2017 8;2(2):024302.

[20] Toschi F, Bodenschatz E. Lagrangian properties of particles in turbulence. *Annual review of fluid mechanics*. 2009;41:375-404.

[21] Brandt L, Coletti F. Particle-laden turbulence: progress and perspectives. *Annual Review of Fluid Mechanics*. 2022;54:159-89.

[22] Yeung PK. Lagrangian investigations of turbulence. *Annual review of fluid mechanics*. 2002 Jan;34(1):115-42.

[23] Maxey MR, Riley JJ. Equation of motion for a small rigid sphere in a nonuniform flow. *The Physics of Fluids*. 1983;26(4):883-9.

[24] Maxey MR. The gravitational settling of aerosol particles in homogeneous turbulence and random flow fields. *Journal of fluid mechanics*. 1987;174:441-65.

[25] Ruetsch GR, Meiburg E. On the motion of small spherical bubbles in two-dimensional vortical flows. *Physics of Fluids A: Fluid Dynamics*. 1993;5(10):2326-41.

[26] Wang LP, Maxey MR. The motion of microbubbles in a forced isotropic and homogeneous turbulence. *Applied Scientific Research*. 1993;51(1):291-6.

[27] Squires KD, Eaton JK. Particle response and turbulence modification in isotropic turbulence. *Physics of Fluids A: Fluid Dynamics*. 1990;2(7):1191-203.

[28] Ruetsch GR, Maxey MR. The evolution of small-scale structures in homogeneous isotropic turbulence. *Physics of Fluids A: Fluid Dynamics*. 1992;4(12):2747-60.

[29] Wang Q, Squires KD. Large eddy simulation of particle-laden turbulent channel flow. *Physics of Fluids*. 1996;8(5):1207-23.

[30] Rouson DW, Eaton JK. On the preferential concentration of solid particles in turbulent channel flow. *Journal of Fluid Mechanics*. 2001;428:149-69.

[31] Monchaux R, Dejoan A. Settling velocity and preferential concentration of heavy particles under two-way coupling effects in homogeneous turbulence. *Physical Review Fluids*. 2017;2(10):104302.

[32] Wang G, Fong KO, Coletti F, Capecelatro J, Richter DH. Inertial particle velocity and distribution in vertical turbulent channel flow: a numerical and experimental comparison. *International Journal of Multiphase Flow*. 2019;120:103105.

[33] Bappy MH, Vela-Martin A, Buscaglia GC, Carrica PM, Freire LS. Effect of bubble size on Lagrangian pressure statistics in homogeneous isotropic turbulence. In *Journal of Physics: Conference Series* 2020;1522(1):012002. IOP Publishing.

[34] Bappy MH, Carrica PM, Vela-Martín A, Freire LS, Buscaglia GC. Pressure statistics of gas nuclei in homogeneous isotropic turbulence with an application to cavitation inception. *Physics of Fluids.* 2020;32(9):095107.

[35] Bappy MH, Carrica PM, Li J, Martin JE, Vela-Martín A, Freire LS, Buscaglia GC. A sub-grid scale cavitation inception model. *Physics of Fluids*. 2022;34(3):033308.

[36] Balachandar S, Eaton JK. Turbulent dispersed multiphase flow. *Annual review of fluid mechanics*. 2010;42:111-33.