The Cosmic Black Hole as a Solution of the Relativistic Quantum Mechanical DIRAC Equation

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Abstract **— For many physicists, Albert Einstein's Theory of General Relativity is the top of Physics. A whole new concept, based on a flexible Space-Time Continuum. A wonderful New and Original insight in the Origins of Space and Time. But nowadays physics requires more than a fundamental theory about Space and Time.**

The Mathematical foundation for a "Quantum Mechanical Model of the Black Hole" is based on a 10-dimensional Space-Time Continuum. This article has been written in projections of a 10-Dimensional Space-Time Continuum within an easier to understand 4-Dimensional Space-Time Continuum. For that reason, this theory will not start with "Einstein's famous Field Equations", but the start will be at a very fundamental concept in Physics. Isaac Newton's 3rd law as a fundament in Classical- and Quantum Mechanics.

To make the theory of the "Quantum Mechanical Model of the Black Hole" as much understandable as possible, this article starts with a short comprehension of the theory.

Keywords **—** Black Hole, General Relativity, Quantum Field Theory, Quantum Gravity.

I. The Fundamental Corrections in "Maxwell's Theory of Electrodynamics" to Build a Framework for a "Quantum Mechanical Model of the Black Hole

To describe a "**Quantum Mechanical Model of the Black Hole**" the Inertia (Mass) of Light has to be included within the Fundamental Electromagnetic Field Equations, which Describes Classical Electrodynamics. Maxwell's Equations do not include an Inertia Term for Light. For that reason a new "Fundamental Electromagnetic Equation" has to be developed to describe the Electromagnetic field (including Inertia and Electromagnetic Interaction) in a different and more complete way than Maxwell did (Einstein called Maxwell the greatest physicist of his century and in that century, Maxwell was).

A. Newton's Approach to Electrodynamics and Electromagnetic Interaction

Newton found the concept of "Universal Equilibrium" which he formulated in his famous third equation Action = - Reaction. In nowadays math the concept of "**Universal Equilibrium**" has been formulated as:

$$\sum_{i=0}^{i=n} \overline{\mathrm{F_i}} = 0 \tag{1}$$

Because the Inertia Force is a Reaction Force, the Inertia Force appears in the equation with a minus sign.

$$\sum_{i=0}^{i=n} \overline{\mathrm{F_i}} - \mathrm{m}\,\bar{\mathrm{a}} = 0 \tag{2}$$

Equation (2) is a general presentation of Newton's famous second law of motion. In a fundamental way, Newton's second law of motion describes the required electromagnetic equation for the Gravitational-Electromagnetic Interaction in general terms, including Maxwell's theory of Electrodynamics published in 1865 in the article: "A Dynamic Theory of the Electromagnetic Field" and Einstein's theory of General Relativity published in 1911 the article: "On the Influence of Gravitation on the Propagation of Light".

Because Maxwell's 4 equations are not part of one whole uniform understanding of the universe like the fundamental equation of Newton's second law of motion represents, Maxwell's theory is missing the fundamental foundation.

Newton's second law of motion has been based on a profound understanding of the universe which is

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based on the fundamental principle of "**Harmony and Equilibrium**", expressed in equation (2).

To realize the new "Gravitational-Electromagnetic Equation", Newton's second law of motion will be the Universal Concept in Physics on which "**Quantum Light Theory**" will be built. The fundamental Electromagnetic force density equation has been based integral on Newton's second law of motion and has been divided into 5 separate terms (B-1 until B-5), each one describing a part of the electromagnetic and inertia force densities.

$$\sum_{i=0}^{i=5} B_i = 0 \tag{3}$$

The first term B-1 represents the inertia of the mass density of light (Electromagnetic Radiation). The terms B-2 and B-3 represent the electric force densities within the Electromagnetic Radiation (Beam of Light) and the terms B-4 and B-5 represent the magnetic force densities within the Electromagnetic Radiation (Beam of Light).

Fundamental in the "Quantum Light Theory" is the outcome of (3) which always has to be zero according Newton's fundamental principle of "Universal Equilibrium".

To apply the concept of "Universal Equilibrium" within an electromagnetic field, the electric forces $F_{Electric}$, the magnetic forces $F_{Magnetic}$ and the inertia forces will be presented separately in equation (3):

$$\sum_{i=0,\, j=0}^{i=n,\, j=m} \left(\overline{F_{Electric\text{-}i}} + \overline{F_{Magnetic-j}} - m\,\bar{a} \right) = 0 \tag{4}$$

B. The First Term in "Quantum Mechanical General Relativity" (Term B-1)

Albert Einstein's "General Relativity" describes the interaction between "Gravity and Light". It is impossible to build a framework for the interaction between "Gravity and Light" without describing the inertia (mass) of light. Because there can only be interaction between gravity and inertia (mass). Without inertia (mass) there will be no interaction with gravity. The mass of light is very different than the mass of objects we usually describe. The mass (inertia) of objects is always omni-directional. It does not matter in what for a position we put a mass on a scale. The weight (interaction) between gravity and the object will always be the same.

That is not the fact for light. A spherical beam of light has no mass. The mass (inertia) will occur when the electromagnetic energy density in the light changes. At the edges of "Light" and "No Light", the interaction between gravity and light will take place. For a LASER beam the interaction between gravity and light will take place at the edges of the LASER beam. For that reason a LASER beam will be deflected by a gravitational field when it pass a large mass. But when a LASER beam propagates towards (or away from) a Back Hole, the speed of light will not change because there is no interaction.

The result of interaction is a "Force". When a beam of light propagates towards the Earth and will be (partly absorbed and partly reflected), the Electromagnetic Energy Flux changes in direction (partly becomes zero, and has been absorbed) and Newton's effect of "F = m a" becomes noticeable in a radiation pressure e.g. on the Earth of a few ton.

Reducing Equation (2) to one single Force $\bar{F}$, equation (2) will be written in the well-known presentation:

$$\bar{F} = m\,\bar{a} \tag{5}$$

The right and the left term of Newton's law of motion in equation (5) has to be divided by the Volume "V" to find an equation for the force density $\bar{f}$ related to the mass density "ρ".

$$\begin{aligned} \bar{F} &= m\,\bar{a} \\ \left(\frac{\bar{F}}{V}\right) &= \left(\frac{m}{V}\right)\bar{a} \\ \bar{f} &= \rho\,\bar{a} \end{aligned} \tag{6}$$

The Inertia Force $\overline{F_{Inertia}}$ for Electromagnetic Radiation will be derived from Newton's second law of motion, using the relationship between the momentum vector $\bar{p}$ for radiation expressed by the Poynting vector $\bar{S}$:

$$\overline{F_{INERTIA}} = -m\,\bar{a} = -m\frac{\Delta\,\bar{v}}{\Delta\,t} = -\frac{\Delta(m\bar{v})}{\Delta\,t} = -\frac{\Delta\,\bar{p}}{\Delta\,t} = -\left(\frac{V}{c^2}\right)\frac{\Delta\bar{S}}{\Delta t} \tag{7}$$

$$\overline{F_{INERTIA}} = -m\,\bar{a} = -m\frac{\Delta\,\bar{v}}{\Delta\,t} = -\frac{\Delta(m\bar{v})}{\Delta\,t} = -\frac{\Delta\,\bar{p}}{\Delta\,t} = -\left(\frac{V}{c^2}\right)\frac{\Delta\bar{S}}{\Delta t}$$

Dividing the right and the left term in equation (7) by the volume V results in the inertia force density $\overline{f_{Inertia}}$:

$$\begin{aligned}\overline{F_{INERTIA}} &= -m\,\bar{a} = -m\frac{\Delta\bar{v}}{\Delta t} = -\frac{\Delta(m\bar{v})}{\Delta\,t} = -\frac{\Delta\bar{p}}{\Delta t} = -\left(\frac{V}{c^2}\right)\frac{\Delta\bar{S}}{\Delta t}\\ \frac{\overline{F_{INERTIA}}}{V} &= -\frac{m}{V}\bar{a} = -\frac{m}{V}\frac{\Delta\,\bar{v}}{\Delta\,t} = -\frac{1}{V}\frac{\Delta\,\bar{p}}{\Delta\,t} = -\left(\frac{1}{c^2}\right)\frac{\Delta\,\bar{S}}{\Delta\,t}\\ \overline{f_{INERTIA}} &= -\rho\,\bar{a} = -\left(\frac{1}{c^2}\right)\frac{\Delta\,\bar{S}}{\Delta\,t}\quad[N/m^3]\end{aligned} \tag{8}$$

The Poynting vector $\bar{S}$ represents the total energy transport of the electromagnetic radiation per unit surface per unit time [J/m^2 s]. Which can be written as the cross product of the Electric Field intensity $\bar{E}$ and the magnetic Field intensity $\bar{H}$.

$$\begin{aligned}\overline{f_{INERTIA}} &= -\rho a = -\left(\frac{1}{c^2}\right)\frac{\Delta\,S}{\Delta\,t} = -\left(\frac{1}{c^2}\right)\frac{\Delta\left(\bar{E}\times\bar{H}\right)}{\Delta\,t}\,[N/m^3]\\ \overline{f_{INERTIA}} &= -\left(\frac{1}{c^2}\right)\frac{\partial\left(\bar{E}\times\bar{H}\right)}{\partial\,t}\,[N/m^3]\end{aligned} \tag{9}$$

C. Coulomb's Law (Colomb Force) for Electromagnetic GEONs (Term B-2 and B-4)

John Archibald Wheeler (a friend of Albert Einstein) introduced in 1955 the concept of **G**ravitational **E**lectr**o**magnetic E**n**tities (GEONs). An example of the Coulomb Force is the Electric Force $F_{Coulomb}$ acting on an electric charge Q placed in an electric field E. The equation for the Coulomb Force equals:

$$\overline{F_{Coulomb}} = \bar{E}\,Q\quad[N] \tag{10}$$

Dividing the right and the left term in equation (10) by the volume V results in the Electric force density $\overline{f_{Coulomb}}$:

$$\begin{aligned}\bar{F}_{COULOMB} &= \bar{E}\,Q\quad[N]\\ \frac{\bar{F}_{COULOMB}}{V} &= \bar{E}\,\frac{Q}{V}\,[N/m^3]\\ \bar{f}_{COULOMB} &= \bar{E}\,\rho_E\,[N/m^3]\end{aligned} \tag{11}$$

Substituting Gauss's law in differential form in (11) results in Coulombs Law for Electromagnetic Radiation for the Electric force density $\overline{f_{Coulomb}}$:

$$\begin{aligned}\bar{f}_{COULOMB} &= \bar{E}\,\rho_E\\ \bar{f}_{COULOMB} &= \bar{E}\,\rho_E = \bar{E}\left(\nabla\,.\,\bar{D}\right)\\ \bar{f}_{COULOMB} &= \bar{E}\left(\nabla\,.\,D\right) = \varepsilon\,\bar{E}\left(\nabla\,.\,\bar{E}\right)\,[N/m^3]\end{aligned} \tag{12}$$

In Electromagnetic Field Configurations, there is in general no preference for the electric force densities or the magnetic force densities. In general the equations for the electric field densities are universally exchangeable with the magnetic field densities.

For the magnetic field densities, equation (12) can be written as:

$$\overline{f}_{\text{Coulomb - Electric}} = \overline{E}\left(\nabla.\,\overline{D}\right) = \varepsilon\,\overline{E}\,\left(\nabla.\,\overline{E}\right)\,\left[N/\,m^{3}\right]\ \text{(Term B-2)} \tag{13}$$
$$\overline{f}_{\text{Coulomb - Magnetic}} = \overline{H}\left(\nabla.\,\overline{B}\right) = \mu\,\overline{H}\,\left(\nabla.\,\overline{H}\right)\,\left[N/\,m^{3}\right]\ \text{(Term B-4)}$$

D. Lorentz's Law (Lorentz Force) for Electromagnetic GEONs (Term B-3 and B-5)

An example of the Lorentz Force is the Magnetic Force $F_{Lorentz}$ acting on an electric charge Q moving with a velocity v within a magnetic field with magnetic field intensity B (magnetic induction).

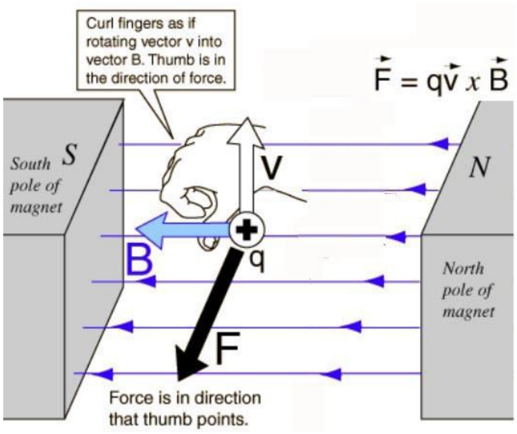

Fig. 1. The Lorentz Force equals the cross product of the Magnetic Induction B and the velocity v of the charge q moving within the magnetic field times the value of the electric charge.

The equation for the Lorentz Force equals:

$$\overline{F}_{\text{LORENTZ}} = Q\,\overline{v} \times \overline{B}\quad [N] \tag{14}$$

Dividing the right and the left term in equation (14) by the volume V results in the Lorentz force density $\overline{f_{Lorentz}}$:

$$\overline{F}_{\text{LORENTZ}} = Q\,\overline{v} \times \overline{B}\quad [N]$$
$$\frac{\overline{F}_{\text{LORENTZ}}}{V} = -\,\overline{B} \times \frac{Q\,\overline{v}}{V}\,\left[N/\,m^{3}\right] \tag{15}$$
$$\overline{f}_{\text{LORENTZ}} = -\,\overline{B} \times \frac{Q\,\overline{v}}{V} = -\,\overline{B} \times \overline{j} = -\,\mu\,\overline{H} \times \overline{j}\,\left[N/\,m^{3}\right]$$

In which q is the electric charge, v the velocity of the electric charge, B the magnetic induction and j the electric current density. Substituting Ampère's law in differential form in (15) results in Lorentz's Law for Electromagnetic Radiation for the Electric force density $\overline{f_{Lorentz}}$:

$$\overline{f}_{\text{LORENTZ}} = -\,\mu\,\overline{H} \times \left(\overline{j}\,\right)$$
$$\overline{f}_{\text{LORENTZ}} = -\,\mu\,\overline{H} \times \left(\overline{j}\,\right) = -\,\mu\,\overline{H} \times \left(\nabla \times \overline{H}\right)\,\left[N/\,m^{3}\right] \tag{16}$$

In Electromagnetic Field Configurations, there is in general no preference for the electric force densities or the magnetic force densities. In general the equations for the electric field densities are universally exchangeable with the magnetic field densities. For the electric field densities, equation (16) can be written as:

$$\overline{f}_{\text{Coulomb - Electric}} = -\,\varepsilon\, \overline{E} \times \left(\nabla \times \overline{E}\right)\left[\text{N/ m}^3\right] \text{ (Term B-3)}$$
$$\overline{f}_{\text{Coulomb - Magnetic}} = -\,\mu\, \overline{H} \times \left(\nabla \times \overline{H}\right)\left[\text{N/ m}^3\right] \text{(Term B-5)} \tag{17}$$

E. The Universal Equation for the Electromagnetic field (Term B-1 + Term B-2 + Term B-3 + Term B-4 + Term B-5)

Newton's second law of motion applied within any arbitrary electromagnetic field configuration results in the fundamental equation (23) for any arbitrary electromagnetic field configuration (a beam of light):

$$\text{NEWTON: } F_{\text{TOTAAL}} = m\,a \text{ represents: } f_{\text{TOTAAL}} = \rho\, a$$
$$-\rho\, a + f_{TOTAAL} = 0$$
$$-\rho\, a + f_{\text{ELEKTRISCH}} + f_{\text{MAGNETISCH}} = 0$$
$$-\rho\, a + F_{\text{COULOMB}} + F_{\text{LORENTZ}} + F_{\text{COULOMB}} + F_{\text{LORENTZ}} = 0 \tag{18}$$
$$-\frac{1}{c^2}\frac{\partial\,(\overline{E}\times\overline{H})}{\partial t} + \varepsilon_0\, \overline{E}(\nabla . \overline{E}) - \varepsilon_0\, \overline{E}\times(\nabla\times\overline{E}) + \mu_0\, \overline{H}\,(\nabla . \overline{H}) - \mu_0\, \overline{H}\times(\nabla\times\overline{H}) = 0$$

B-1 B-2 B-3 B-4 B-5

Term B-4 is the magnetic equivalent of the (electric) Coulomb's law B-2 and Term B-3 is the electric equivalent of the (magnetic) Lorentz's law B-5.

The universal equation for the electromagnetic field (free electromagnetic waves and confined electromagnetic fields) has been presented in (24) and expresses the perfect equilibrium between the inertia forces (B-1), the electric forces (B-2 and B-3) and the magnetic forces (B-4 and B-5) in any arbitrary electromagnetic field configuration.

$$-\frac{1}{c^2}\frac{\partial\,(\overline{E}\times\overline{H})}{\partial t} + \varepsilon_0\, \overline{E}(\nabla . \overline{E}) - \varepsilon_0\, \overline{E}\times(\nabla\times\overline{E}) + \mu_0\, \overline{H}\,(\nabla . \overline{H}) - \mu_0\, \overline{H}\times(\nabla\times\overline{H}) = 0 \tag{19}$$

B-1 B-2 B-3 B-4 B-5

F. The Universal Integration of Maxwell's Theory of Electrodynamics

The universal equation (19) for any arbitrary electromagnetic field configuration can be written in the form:

$$-\frac{1}{c^2}\frac{\partial\,(\overline{E}\times\overline{H})}{\partial t} + \varepsilon_0\, \overline{E}(\nabla . \overline{E}) - \varepsilon_0\, \overline{E}\times(\nabla\times\overline{E}) + \mu_0\, \overline{H}\,(\nabla . \overline{H}) - \mu_0\, \overline{H}\times(\nabla\times\overline{H}) = 0$$
$$-\varepsilon_0\mu_0\left(\overline{E}\times\frac{\partial(\overline{H})}{\partial t} + \overline{H}\times\frac{\partial(\overline{E})}{\partial t}\right) + \varepsilon_0\,\overline{E}(\nabla . \overline{E}) - \varepsilon_0\,\overline{E}\times(\nabla\times\overline{E}) + \mu_0\,\overline{H}\,(\nabla . \overline{H}) - \mu_0\,\overline{H}\times(\nabla\times\overline{H}) = 0 \tag{20}$$
$$-\left(\varepsilon_0\overline{E}\times\frac{\partial\,(\overline{B})}{\partial t} + \mu_0\overline{H}\times\frac{\partial\,(\overline{D})}{\partial t}\right) + \overline{E}(\nabla . \overline{D}) - \varepsilon_0\,\overline{E}\times(\nabla\times\overline{E}) + \overline{H}\,(\nabla . \overline{B}) - \mu_0\,\overline{H}\times(\nabla\times\overline{H}) = 0$$

M-3 M-4 M-1 M-3 M-2 M-4

The Maxwell Equations are presented in (21):

$$\nabla . \overline{D} = \rho \quad \text{(M-1)} \qquad \nabla\times\overline{E} = -\frac{\partial B}{\partial t} \quad \text{(M-3)}$$
$$\nabla . \overline{B} = 0 \quad \text{(M-2)} \qquad \nabla\times\overline{H} = \frac{\partial D}{\partial t} \quad \text{(M-4)} \tag{21}$$

In vacuum in the absence of any charge density, it follows from (26) that all the solutions for the Maxwell's Equations are also solutions for the separate parts of the Universal Equation (25) for the Electromagnetic field.

Universal Equation for the Electromagnetic Field.

$$-\left(\varepsilon_0 \overline{E} \times \frac{\partial (\overline{B})}{\partial t} + \mu_0 \overline{H} \times \frac{\partial (\overline{D})}{\partial t}\right) + \overline{E}(\nabla . \overline{D}) - \varepsilon_0 \overline{E} \times (\nabla \times \overline{E}) + \overline{H}(\nabla . \overline{B}) - \mu_0 \overline{H} \times (\nabla \times \overline{H}) = 0$$

M-3 M-4 M-1 M-3 M-2 M-4

4 Maxwell's Equations (22)

$$\nabla . \overline{D} = \rho \quad \text{(M-1)} \qquad \nabla \times \overline{E} = -\frac{\partial B}{\partial t} \quad \text{(M-3)}$$

$$\nabla . \overline{B} = 0 \quad \text{(M-2)} \qquad \nabla \times \overline{H} = \frac{\partial D}{\partial t} \quad \text{(M-4)}$$

Comparing the 4 Maxwell Equations (26) with the Universal Equation (24) we conclude that the 4 Maxwell equations show only the 4 parts of the Universal Dynamic Equilibrium in 4 separate terms and the 4 Maxwell equations are missing the fundamental term for inertia. For that reason it is impossible to calculate the interaction between light and gravity with the 4 Maxwell equations. To find the interaction terms between light and gravity the inertia term (B-1 in 24) is necessary.

II. The Gravitational-Electromagnetic Confinement within a Black Hole

"John Archibald Wheeler", published his theory about **G**ravitational **E**lectr**o**magnetic E**n**titie**s** (GEONs) in 1955 in Physical Review Journals: "GEONs" [1]. John Archibald Wheeler was the first one in physics who understood that:

$$\textit{Light} \triangleq \textit{Electromagnetic Radiation} \triangleq \textit{Energy} \triangleq$$
$$\triangleq \textit{Mass}\ \left(E = m\,c^2\right) \triangleq \textit{Gravity} \triangleq \textit{Gravitational Field}$$

Wheeler's conclusion was that every beam of light creates simultaneously a gravitational field, besides the original electromagnetic field, proportional to the electromagnetic energy of the beam of light.

Like a Black Hole can capture a beam of light, it is possible that a beam of light confines itself due to its own gravitational field forming a **G**ravitational **E**lectr**o**magnetic E**n**tity (GEON). The final results of Wheeler's research were not what he expected. He found only GEONs by toroidal "Gravitational-Electromagnetic Confinement" with the dimensions of a star constellation and a stable lifetime of a few milliseconds.

"Quantum Light Theory" presents a new "Gravitational-Electromagnetic Equation" [7], [8] and [9] describing Electromagnetic Field Configurations which are simultaneously the Mathematical Solutions for the Quantum Mechanical "Schrodinger Wave Equation" and more exactly the Mathematical Solutions for the "Relativistic Quantum Mechanical Dirac Equation". The Mathematical Solutions for the "Gravitational-Electromagnetic Equation" carry Mass, Electric Charge and Magnetic Spin at discrete values.

When a beam of light is approaching a strong gravitational field in the direction of the gravitational field, generated by a Black Hole, the confinement has been called a Longitudinal Black Hole. The direction of propagation of the beam of light is in the same direction (or in the opposite direction) of the gravitational field. According the first term in (33), the beam of light will be accelerated or decelerated. However, the speed of light is a universal constant and for that reason the speed of light cannot increase or decrease. Instead the intensity of the electromagnetic radiation will increase when the beam of light approaches (propagates in the opposite direction as the direction of the gravitational field) the Black Hole. And the intensity of the electromagnetic radiation will decrease when the beam of light leaves (propagates in the same direction as the direction of the gravitational field) the Black Hole.

The Gravitational-Electromagnetic Confinement for the elementary structure beyond the "superstring" / "Black Hole" is presented in equation (49).

3-Dimensional Space Domain

$$\begin{pmatrix} x_3 \\ x_2 \\ x_1 \end{pmatrix} \quad -\frac{1}{c^2}\frac{\partial\,(\overline{E}\times\overline{H})}{\partial t} + \varepsilon_0\,\overline{E}\,(\nabla\,.\,\overline{E}) - \varepsilon_0\,\overline{E}\times(\nabla\times\overline{E}) + \mu_0\,\overline{H}\,(\nabla\,.\,\overline{H}) - \mu_0\,\overline{H}\times(\nabla\times\overline{H}) + \frac{1}{2}\left(\varepsilon^2\mu\left(\overline{E}\bullet\overline{E}\right)+\varepsilon\,\mu^2\left(\overline{H}\bullet\overline{H}\right)\right)\overline{g} = \overline{0} \tag{23}$$

In which $\overline{g}$ represents the gravitational acceleration acting on the electromagnetic mass density of the confined electromagnetic radiation.

A possible solution for equation (49) describing an **G**ravitational **E**lectr**o**mag**n**etic Confinement (GEON) within a radial gravitational field with acceleration $\overline{g}$ has been represented in (50).

$$\begin{pmatrix} e_r \\ e_\theta \\ e_\varphi \end{pmatrix} = \begin{pmatrix} 0 \\ f(r)\,\mathrm{Sin}(\omega\,t) \\ -f(r)\,\mathrm{Cos}(\omega\,t) \end{pmatrix} \quad \begin{pmatrix} m_r \\ m_\theta \\ m_\varphi \end{pmatrix} = \begin{pmatrix} 0 \\ f(r)\,\mathrm{Cos}(\omega\,t) \\ f(r)\,\mathrm{Sin}(\omega\,t) \end{pmatrix} \quad \overline{g} = \begin{pmatrix} \frac{G_1}{4\,\pi\,r^2} \\ 0 \\ 0 \end{pmatrix} \tag{24}$$

$$w_{em} = \left(\frac{\mu_0}{2}\left(\overline{m}\,.\,\overline{m}\right)+\frac{\varepsilon_0}{2}\left(\overline{e}\,.\,\overline{e}\right)\right) = \varepsilon_0\,f(r)^2$$

In which the radial function f(r) equals:

$$f[r] = K\,\mathrm{e}^{-\frac{-\frac{G1\,\varepsilon_0\,\mu_0}{r}+8\,\pi\,\log[r]}{8\,\pi}} \tag{25}$$

The solution has been calculated according Newton's Shell Theorem.

A. Black Hole at atomic scale with an electromagnetic mass: e_{mm} = 1.6726×10^{-27} [kg] and the radius = 3×10^{-58} [m]

For an electromagnetic mass of the confinement: e_{mm} = 1.6726×10^{-27} [kg] (mass of proton), the radius of the confinement equals approximately 3×10^{-58} [m]. This is far beyond the order of Planck's Length.

The Plot graph of the Electric Field Intensity f(r) of the confinement has been presented as a function of the radius in Fig. 2 and Fig. 3:

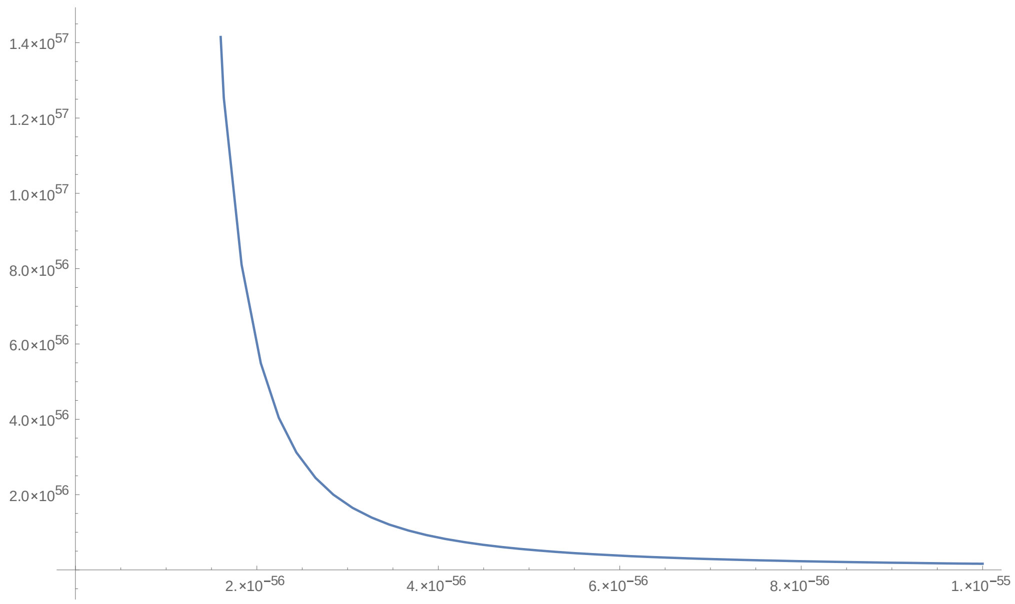

Fig. 2. PlotGraph of the Electric Field Intensity f(r) [V/ m] for the region 10^{-59} < r < 10^{-55} [m] in which the gravitational field acceleration has been chosen accordingly an electromagnetic mass of $1.6\,726\times10^{-27}$ [kg] located at the center of the confinement, according Newton's Shell Theorem.

$$Plot\left[e^{-\frac{-\frac{G1\,\varepsilon_0\,\mu_0}{r}+8\,\pi\,\log[r]}{8\,\pi}},\{r,10^{-59},10^{-57}\}\right]$$

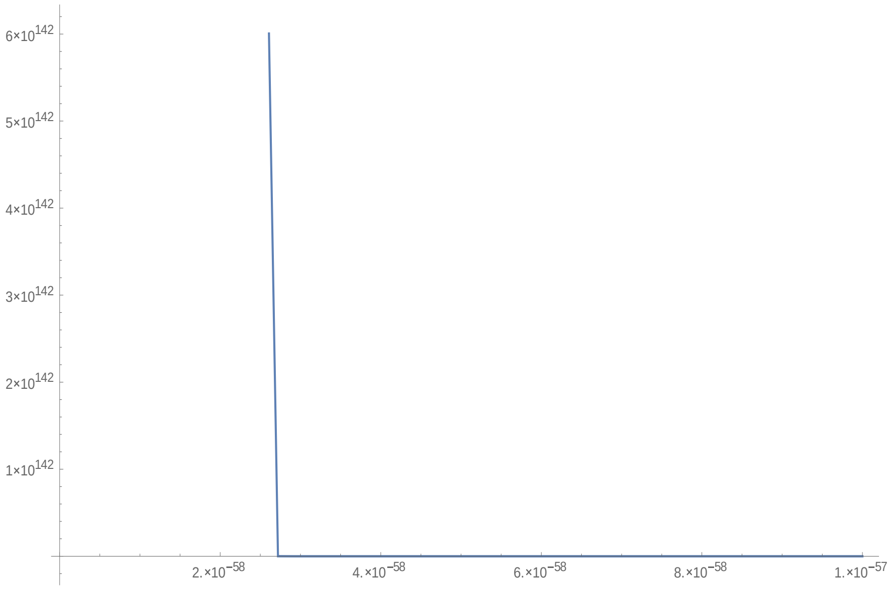

Fig. 3. PlotGraph of the Electric Field Intensity f(r) [V/m] for the region $10^{-59} < r < 10^{-57}$ [m] in which the gravitational field acceleration has been chosen accordingly an electromagnetic mass of 1.6726 x 10^{-27} [kg] located at the center of the confinement, according Newton's Shell Theorem.

The fundamental question is: How it is possible to create confinements from "visible light" (with a wave length between 3.9×10^{-7} [m] until 7×10^{-7} [m]) within dimensions smaller than Planck's Length?

This is only possible when the wave length of the confined radiation is smaller than de dimensions of the confinement. This requires extreme high frequencies. The transformation in frequency from visible light into the extreme high frequency of the confinement is possible because of the Lorentz/ Doppler transformation during the collapse of the radiation when the confinement has been formed (implosion of visible light).

B. The Electric Charge and the Magnetic Spin for "Black Holes"

The following functions for Electromagnetic GEONs with the quantum variables $\{m_1, n_1, p_1, q_1\}$ have been chosen:

$$f[r]=K\,e^{-\frac{-\frac{G1\,\varepsilon_0\,\mu_0}{r}+8\,\pi\,\log[r]}{8\,\pi}}$$

$$g_1(\theta,\varphi,t) = \sin(t\,\omega)\,(\sin(\pi\,\theta\,m_1)\sin(n_1\,2\,\pi\,\varphi)+1)$$

$$h(\theta,\varphi) = \sin(\pi\,\theta\,p_1)\,\sin(q_1\,2\,\pi\,\varphi)+1$$

$$g_2(\theta,\varphi,t) = \frac{\sec(t\,\omega)\sqrt{\cos(2\,t\,\omega)\,g_1(\theta,\varphi,t)^2 - g_1(\theta,\varphi,t)^2 + 2\,h(\theta,\varphi)}}{\sqrt{2}} \tag{26}$$

$$f_1[r,\theta,\varphi,t] = e^{-\frac{-\frac{G1\,\varepsilon_0\,\mu_0}{r}+8\,\pi\,\log[r]}{8\,\pi}}\,K\,g_1[\theta,\varphi,t]$$

$$f_2[r,\theta,\varphi,t] = \frac{e^{-\frac{-\frac{G1\,\varepsilon_0\,\mu_0}{r}+8\,\pi\,\log[r]}{8\,\pi}}\,K\sqrt{-g_1[\theta,\varphi,t]^2+\cos[2\,t\omega]\,g_1[\theta,\varphi,t]^2+2h[\theta,\varphi]}}{\sqrt{2}}$$

C. Black Hole with quantum numbers: (Electric- and Magnetic Dipoles, Electric- and Magnetic Spin) $\{m_1=0, n_1=0, p_1=0, q_1=0\}$

The divergence of the electric field intensity (electric charge density) equals:

$$\nabla.\begin{pmatrix} e_r \\ e_\theta \\ e_\varphi \end{pmatrix} = \frac{\sqrt{2}\ \mathrm{K1}\ \cot(\theta)\ \sin^2(t\ \omega)\ \sqrt{1 - \sin^4(t\ \omega)}\ \mathrm{e}^{\frac{G_1\ \varepsilon_0\ \mu_0}{8\ \pi\ \mathrm{r}}}}{r^2\ \sqrt{2 - 2\sin^4(t\omega)}}$$

$$\nabla.\begin{pmatrix} e_r \\ e_\theta \\ e_\varphi \end{pmatrix} = \frac{\frac{1}{2}\ \mathrm{K1}\ \cot(\theta)\ \mathrm{e}^{\frac{G_1\ \varepsilon_0\ \mu_0}{8\ \pi\ \mathrm{r}}}}{r^2}\ \text{(averaged over 1 period of time)} \tag{27}$$

The divergence of the magnetic field intensity (magnetic monopole) equals:

$$\nabla.\begin{pmatrix} m_r \\ m_\theta \\ m_\varphi \end{pmatrix} = \frac{\mathrm{K1}\ \sqrt{\varepsilon_0}\ \cot(\theta)\ \sqrt{2 - 2\ \sin^4(t\ \omega)}\quad \mathrm{e}^{\frac{G_1\ \varepsilon_0\ \mu_0}{8\ \pi\ \mathrm{r}}}}{\sqrt{2}\ \sqrt{\mu 0}\ r^2}$$

$$\nabla.\begin{pmatrix} m_r \\ m_\theta \\ m_\varphi \end{pmatrix} = \frac{\mathrm{K1}\ \sqrt{\varepsilon_0}\ \cot(\theta)\ \sqrt{\frac{3}{4}}\quad \mathrm{e}^{\frac{G_1\ \varepsilon_0\ \mu_0}{8\ \pi\ \mathrm{r}}}}{\sqrt{\mu 0}\ r^2}\ \text{(averaged over 1 period of time)} \tag{28}$$

In which K1 is an arbitrary variable. Because of the $\mathrm{Cot}(\theta)$ function, the electric divergence as well as the magnetic divergence changes from sign when the angle θ varies between 0º until 360º forming electric dipoles (+ versus -) and magnetic dipoles (N versus S).

D. Black Hole with quantum numbers: (Electric- and Magnetic Dipoles, Electric- and Magnetic Spin) {m1=1, n1=0, p1=0, q1=0}

The divergence of the electric field intensity (electric charge density) equals:

$$\nabla.\begin{pmatrix} e_r \\ e_\theta \\ e_\varphi \end{pmatrix} = \frac{\sqrt{2}\ \mathrm{K1}\ \cot(\theta)\ \sin^2(t\omega)\ \sqrt{1 - \sin^4(t\omega)}\ \mathrm{e}^{\frac{G_1\ \varepsilon_0\ \mu_0}{8\ \pi\ \mathrm{r}}}}{r^2\sqrt{2 - 2\ \sin^4(t\omega)}}$$

$$\nabla.\begin{pmatrix} e_r \\ e_\theta \\ e_\varphi \end{pmatrix} = \frac{\frac{1}{2}\mathrm{K1}\ \cot(\theta)\quad \mathrm{e}^{\frac{G_1\ \varepsilon_0\ \mu_0}{8\ \pi\ \mathrm{r}}}}{r^2}\ \text{(averaged over 1 period of time)} \tag{29}$$

The divergence of the magnetic field intensity (magnetic monopole) equals:

$$\nabla.\begin{pmatrix} m_r \\ m_\theta \\ m_\varphi \end{pmatrix} = \frac{\mathrm{K1}\ \sqrt{\varepsilon_0}\ \cot(\theta)\ \sqrt{2 - 2\ \sin^4(t\omega)}\ \mathrm{e}^{\frac{G_1\ \varepsilon_0\ \mu_0}{8\ \pi\ \mathrm{r}}}}{\sqrt{2}\ \sqrt{\mu_0}\ r^2}$$

$$\nabla.\begin{pmatrix} m_r \\ m_\theta \\ m_\varphi \end{pmatrix} = \frac{\mathrm{K1}\ \sqrt{\frac{3}{4}}\ \sqrt{\varepsilon_0}\ \cot(\theta)\ \mathrm{e}^{\frac{G_1\ \varepsilon_0\ \mu_0}{8\ \pi\ \mathrm{r}}}}{\sqrt{\mu_0}\ r^2}\ \text{(averaged over 1 period of time)} \tag{30}$$

In which K1 is an arbitrary variable. Because of the $\mathrm{Cot}(\theta)$ function, the electric divergence as well as the magnetic divergence changes from sign when the angle θ varies between 0º until 360º forming electric dipoles (+ versus -) and magnetic dipoles (N versus S).

E. Black Hole with quantum numbers: (Electric- and Magnetic Dipoles, Electric- and Magnetic Spin) {m_1=0, n_1=0, p_1=1, q_1=0}

The divergence of the electric field intensity (electric charge density) equals:

$$\nabla.\begin{pmatrix} e_r \\ e_\theta \\ e_\varphi \end{pmatrix} = \frac{e^{\frac{G_1\,\varepsilon_0\,\mu_0}{8\,\pi\,r}}\,K1\cot[\theta]\sin[t\omega]^2}{r^2}$$

$$\nabla.\begin{pmatrix} e_r \\ e_\theta \\ e_\varphi \end{pmatrix} = \frac{e^{\frac{G_1\,\varepsilon_0\,\mu_0}{8\,\pi\,r}}\,K1\cot[\theta]}{2\,r^2} \text{ (averaged over 1 period of time)} \tag{31}$$

The divergence of the magnetic field intensity (magnetic monopole) equals:

$$\nabla.\begin{pmatrix} m_r \\ m_\theta \\ m_\varphi \end{pmatrix} = \frac{e^{\frac{G_1\,\varepsilon_0\,\mu_0}{8\,\pi\,r}}\,K1\,\sqrt{\varepsilon_0}\,\cot[\theta]\,\sqrt{1-\sin[t\omega]^4}}{r^2\,\sqrt{\mu_0}}$$

$$\nabla.\begin{pmatrix} m_r \\ m_\theta \\ m_\varphi \end{pmatrix} = \frac{e^{\frac{G_1\,\varepsilon_0\,\mu_0}{8\,\pi\,r}}\,K1\,\sqrt{\varepsilon_0}\,\cot[\theta]}{2\,r^2\,\sqrt{\mu_0}} \text{ (averaged over 1 period of time)} \tag{32}$$

In which K1 is an arbitrary variable. Because of the $\mathrm{Cot}(\theta)$ function, the electric divergence as well as the magnetic divergence changes from sign when the angle θ varies between 0º until 360º forming electric dipoles (+ versus -) and magnetic dipoles (N versus S).

III. The Mathematical Foundation for the Physical Boundaries of Black Hole as a Solution of the Relativistic Quantum Mechanical Dirac Equation

Newton's second law can be interpreted as the law of "Conservation of Energy". For an Electromagnetic Field the law of conservation of Energy has been expressed as:

$$\nabla\,.\,\bar{\mathrm{S}} + \frac{\partial\,\mathrm{w}}{\partial\,t} = 0 \tag{33}$$

From the equation for the "Conservation of Electromagnetic Energy" the Relativistic Quantum Mechanical "Dirac" equation will be derived which can be considered to be the relativistic version of the Quantum Mechanical Schrodinger wave equation.

A. *The Fundamental Concept of Probability*

The idea of complex (probability) waves is directly related to the concept of confined (standing) waves. Characteristic for any standing wave is the fact that the velocity and the pressure (electric field and magnetic field) are always shifted over 90 degrees. The same principle does exist for the standing (confined) electromagnetic waves,

For that reason every confined (standing) Electromagnetic wave can be described by a complex sum vector $\bar{\phi}$ of the Electric Field Vector $\bar{\mathrm{E}}$ and the Magnetic Field Vector $\bar{\mathrm{B}}$ ($\bar{\mathrm{E}}$ has 90 degrees phase shift compared to $\bar{\mathrm{B}}$).

The vector functions $\bar{\phi}$ and the complex conjugated vector function $\bar{\phi}*$ will be written as:

$$\bar{\phi} = \frac{1}{\sqrt{2\,\mu}}\left(\bar{\mathrm{B}} + \mathrm{i}\,\frac{\bar{\mathrm{E}}}{c}\right) \tag{34}$$

$\bar{\mathrm{B}}$ equals the magnetic induction, $\bar{\mathrm{E}}$ the electric field intensity ($\bar{\mathrm{E}}$ has + 90 degrees phase shift compared to $\bar{\mathrm{B}}$) and c the speed of light.

The complex conjugated vector function equals:

$$\overline{\phi^*} = \frac{1}{\sqrt{2\,\mu}}\left(\overline{B} - i\,\frac{\overline{E}}{c}\right) \tag{35}$$

The dot product equals the electromagnetic energy density w:

$$\overline{\phi}\,.\,\overline{\phi^*} = \frac{1}{2\,\mu}\left(\overline{B} + i\,\frac{\overline{E}}{c}\right).\left(\overline{B} - i\,\frac{\overline{E}}{c}\right) = \frac{1}{2}\,\mu\,H^2 + \frac{1}{2}\,\varepsilon\,E^2 = w \tag{36}$$

Using Einstein's equation $W = m\,c^2$,the dot product equals the electromagnetic mass density w:

$$\overline{\phi}\,.\,\overline{\phi^*}\frac{1}{c^2} = \frac{\varepsilon}{2}\left(\overline{B} + i\,\frac{\overline{E}}{c}\right).\left(\overline{B} - i\,\frac{\overline{E}}{c}\right) = \frac{1}{2}\varepsilon\,\mu^2H^2 + \frac{1}{2}\varepsilon^2E^2 = \rho\ [\mathrm{kg/m^3}] \tag{37}$$

The cross product is proportional to the Poynting vector [3] (page 202, equation 15).

$$\overline{\phi} \times \overline{\phi^*} = \frac{1}{2\,\mu}\left(\overline{B} + i\,\frac{\overline{E}}{c}\right) \times \left(\overline{B} - i\,\frac{\overline{E}}{c}\right) = i\sqrt{\varepsilon\,\mu}\ \overline{E} \times \overline{H} = i\sqrt{\varepsilon\,\mu}\ \overline{S} \tag{38}$$

Newton's second law of motion has been described in 3 spatial dimensions, resulting in the fundamental equation for the electromagnetic field.

3-Dimensional Space Domain

B-1 B-2 B-3 B-4 B-5 B-6

$$\begin{pmatrix} x_3 \\ x_2 \\ x_1 \end{pmatrix} \quad \underbrace{-\frac{1}{c^2}\frac{\partial\,(\overline{E}\times\overline{H})}{\partial t}}_{B-1} + \underbrace{\varepsilon_0\,\overline{E}\,(\nabla\,.\,\overline{E})}_{B-2} - \underbrace{\varepsilon_0\,\overline{E}\times(\nabla\times\overline{E})}_{B-3} + \underbrace{\mu_0\,\overline{H}\,(\nabla\,.\,\overline{H})}_{B-4} - \underbrace{\mu_0\,\overline{H}\times(\nabla\times\overline{H})}_{B-5} + \underbrace{\frac{1}{2}\left(\varepsilon^2\mu\left(\overline{E}\bullet\overline{E}\right) + \varepsilon\,\mu^2\left(\overline{H}\bullet\overline{H}\right)\right)\overline{g}}_{B-6} = \overline{0} \tag{39}$$

The formal mathematical way to describe the force density results from the 4-dimensional divergence of the 4-dimensional energy momentum tensor, resulting in a 4-dimensional Force vector. Dividing the 4-dimensional Force vector by the Volume results in the 4-dimensional force density vector.

The 4-dimensional Electromagnetic Vector Potential has been defined by:

$$\overline{\varphi}^4 = \begin{pmatrix} \varphi_4 \\ \varphi_3 \\ \varphi_2 \\ \varphi_1 \end{pmatrix} \xrightarrow{\text{CartesianCoordinateSystem}} \begin{pmatrix} \varphi_t \\ \varphi_z \\ \varphi_y \\ \varphi_x \end{pmatrix} \tag{40}$$

In which the term φ_a represents the 4-dimensional electromagnetic vector potential in the "a" direction while the indice "a" varies from 1 to 4. In a cartesian coordinate system the indices are chosen varying from the x,y,z and t direction. In which the indice "t" represents the time direction which has been considered to be the 4th dimension. The 4-dimensional Electromagnetic "Maxwell Tensor" has been defined by:

$$F_{ab} = \partial_b\,\varphi_a - \partial_a\,\varphi_b \tag{41}$$

where the indices "a" and "b" vary from 1 to 4.

The 4-dimensional Electromagnetic "Energy Momentum Tensor" has been defined by:

$$T^{ab} = \frac{1}{\mu_0}\left[F_{ac}\,F^{cb} + \frac{1}{4}\delta_{ab}\,F_{cd}\,F^{cd}\right] \tag{42}$$

The 4-dimensional divergence of the 4-dimensional Energy Momentum Tensor equals the 4-dimensional

Force Density 4-vector f^{a} :

$$f^{a} = \partial_{b}\, T^{ab} \tag{43}$$

Substituting the electromagnetic values for the electric field intensity "E" and the magnetic field intensity "H" in (71) results in the 4-dimensional representation of Newton's second law of motion:

Energy-Time Domain

B-7

$$(f_4) \qquad \nabla \cdot (\bar{E} \times \bar{H}) + \frac{1}{2}\frac{\partial\left(\varepsilon_0 \left(\bar{E} \cdot \bar{E}\right) + \mu_0 \left(\bar{H} \cdot \bar{H}\right)\right)}{\partial t} = 0$$

(44)

3-Dimensional Space Domain

B-1 B-2 B-3

B-4 B-5

$$\begin{pmatrix} f_3 \\ f_2 \\ f_1 \end{pmatrix} \quad -\frac{1}{c^2}\frac{\partial (\bar{E} \times \bar{H})}{\partial t} + \varepsilon_0 \bar{E} (\nabla \cdot \bar{E}) - \varepsilon_0 \bar{E} \times (\nabla \times \bar{E}) + \mu_0 \bar{H} (\nabla \cdot \bar{H}) - \mu_0 \bar{H} \times (\nabla \times \bar{H}) = \bar{0}$$

In which f_1 , f_2 , f_3, represent the force densities in the 3 spatial dimensions and f_4 represent the force density (energy flow) in the time dimension (4th dimension). Equation (42) can be written as:

Energy-Time Domain

Inner Energy

B-7

$$(f_4) \qquad \nabla \cdot \bar{S} + \frac{\partial w}{\partial t} = 0 \tag{35.1}$$

3-Dimensional Space Domain

(45)

B-1 B-2 B-3

B-4 B-5

$$\begin{pmatrix} f_3 \\ f_2 \\ f_1 \end{pmatrix} \quad -\frac{1}{c^2}\frac{\partial (\bar{E} \times \bar{H})}{\partial t} + \varepsilon_0 \bar{E} (\nabla \cdot \bar{E}) - \varepsilon_0 \bar{E} \times (\nabla \times \bar{E}) + \mu_0 \bar{H} (\nabla \cdot \bar{H}) - \mu_0 \bar{H} \times (\nabla \times \bar{H}) = \bar{0} \tag{35.2}$$

The 4th term in equation (42) can be written in the terms of the Poynting vector "S" and the energy density "w" representing the electromagnetic law for the conservation of energy (Newton's second law of motion).

B. The 4-Dimensional Relativistic Quantum Mechanical Dirac Equation

Substituting (27) and (28) in Equation (35.1) results in The 4-Dimensional Equilibrium Equation (36):

$$(x_4) \qquad -\frac{i}{\sqrt{\varepsilon_0\, \mu_0}} \nabla \cdot (\bar{\phi} \times \bar{\phi}) = -\frac{\partial \bar{\phi} \cdot \bar{\phi}^*}{\partial t} \tag{36.1}$$

(46)

$$\begin{pmatrix} x_3 \\ x_2 \\ x_1 \end{pmatrix} \quad -\frac{1}{c^2}\frac{\partial (\bar{E} \times \bar{H})}{\partial t} + \varepsilon_0 \bar{E} (\nabla \cdot \bar{E}) - \varepsilon_0 \bar{E} \times (\nabla \times \bar{E}) + \mu_0 \bar{H} (\nabla \cdot \bar{H}) - \mu_0 \bar{H} \times (\nabla \times \bar{H}) = \bar{0} \tag{36.2}$$

To transform the electromagnetic vector wave function $\bar{\phi}$ into a scalar (spinor or one-dimensional matrix representation), the Pauli spin matrices σ and the following matrices [3] (page 213, equation 99) are introduced:

$$\bar{\alpha} = \begin{bmatrix} 0 & \sigma \\ \sigma & 0 \end{bmatrix} \quad \text{and} \quad \bar{\beta} = \begin{bmatrix} \delta_{ab} & 0 \\ 0 & -\delta_{ab} \end{bmatrix} \tag{47}$$

Then equation (44) can be written as the 4-Dimensional Hyperspace Equilibrium Dirac Equation:

$$(x_4) \quad \left(\frac{i\,m\,c}{h}\,\bar{\beta} + \bar{\alpha}\,.\,\nabla\right)\psi = -\frac{1}{c}\,\frac{\partial \psi}{\partial t} \qquad (38.1)$$

$$\begin{pmatrix} x_3 \\ x_2 \\ x_1 \end{pmatrix} \quad -\frac{1}{c^2}\frac{\partial\,(\bar{E}\times\bar{H})}{\partial t} + \varepsilon_0\,\bar{E}\,(\nabla\,.\,\bar{E}) - \varepsilon_0\,\bar{E}\times(\nabla\times\bar{E}) + \mu_0\,\bar{H}\,(\nabla\,.\,\bar{H}) - \mu_0\,\bar{H}\times(\nabla\times\bar{H}) = \bar{0} \qquad (38.2)$$

(48)

The fourth term (x_4) equals the relativistic Dirac equation (38.1) which equals equation (102) page 213 in [3].

Equation (38.1) represents the relativistic quantum mechanical Dirac Equation where ψ represents the quantum mechanical probability wave function. The mathematical evidence for the equivalent for (38.1) has been published in 1995 in the article: "A Continuous Model of Matter based on AEONs". Equation (1) page 201 to Equation (102) page 213. (Doi: 10.31219/osf.io/ra7ng).

The Electromagnetic Law for the conservation of Energy (33) and the Relativistic Dirac Equation (49) and (50) are **identical** but written in a different form.

The law of conservation of Electromagnetic Energy can be written in an electromagnetic form (49) or in an identical way in a quantum mechanical form (50):

Energy-Time Domain
Inner Energy
B-7

$$(f_4) \quad \nabla\,.\,(\bar{\phi}\times\bar{\phi}) = -\frac{i}{c}\,\frac{\partial\,\bar{\phi}\,.\,\bar{\phi}^*}{\partial t} \qquad (39.1)$$

$$(x_4) \quad \left(\frac{i\,m\,c}{h}\,\bar{\beta} + \bar{\alpha}\,.\,\nabla\right)\psi = -\frac{1}{c}\,\frac{\partial \psi}{\partial t} \qquad (39.2)$$

(49)

The weakness in the Quantum Mechanical Relativistic Dirac Equation (39.2) is that the Dirac Equation is a 1-dimensional equation which will never be able to describe the 4-dimensional real physical world. While 39.1 represents a ElectromagneticVector Equation.

From the equations (27) and (28) follows the 4-Dimensional Vector-Dirac equation (40). This equation is a 4-dimensional vector equation and is coherent with the 4-dimensional physical reality.

$$(x_4) \quad \nabla\,.\,(\bar{\phi}\times\bar{\phi}) = -\frac{i}{c}\,\frac{\partial\,\bar{\phi}\,.\,\bar{\phi}^*}{\partial t}$$

$$\begin{pmatrix} x_3 \\ x_2 \\ x_1 \end{pmatrix} \quad \frac{i}{c}\,\frac{\partial\,(\bar{\phi}\times\bar{\phi}^*)}{\partial t} - \left(\bar{\phi}\times(\nabla\times\bar{\phi}^*) + \bar{\phi}^*\times(\nabla\times\bar{\phi})\right) + \left(\bar{\phi}\,(\nabla\,.\,\bar{\phi}^*) + \bar{\phi}^*\,(\nabla\,.\,\bar{\phi})\right) = 0$$

(50)

In which the Quantum Mechanical Complex Probability Vector Function $\bar{\phi}$ and the complex conjugated vector function $\bar{\phi}^*$ equals:

$$\begin{aligned} \overline{\phi} &= \overline{\mathrm{B}} + \frac{i}{c}\overline{\mathrm{E}} = \mu\,\overline{\mathrm{H}} + \frac{i}{c}\overline{\mathrm{E}} \\ \overline{\phi*} &= \overline{\mathrm{B}} - \frac{i}{c}\overline{\mathrm{E}} = \mu\,\overline{\mathrm{H}} - \frac{i}{c}\overline{\mathrm{E}} \end{aligned} \tag{51}$$

The 4-Dimensional Dirac equation represents the "Newtonian Perfect Equilibrium" in the 4-Dimensional Space-Time Continuum en has been represented by 4 separate equations. The first one represents the well-known relativistic quantum mechanical Dirac Equation in the Time-Energy domain x_4. The 3 quantum mechanical equations in the space-momentum domain represents the "Newtonian Perfect Equilibrium" for the force densities in the domains (x_1 ,x_2 ,x_3)

$$(x_4) \qquad \nabla \cdot (\overline{\phi} \times \overline{\phi}) = -\frac{i}{c}\frac{\partial\, \overline{\phi} \cdot \overline{\phi*}}{\partial t}$$

$$\begin{pmatrix} x_3 \\ x_2 \\ x_1 \end{pmatrix} \quad \frac{i}{c}\frac{\partial\,(\overline{\phi} \times \overline{\phi*})}{\partial t} - \left(\overline{\phi} \times (\nabla \times \overline{\phi*}) + \overline{\phi*} \times (\nabla \times \overline{\phi}) \right) + \left(\overline{\phi}\; (\nabla \cdot \overline{\phi*}) + \overline{\phi*}\; (\nabla \cdot \overline{\phi}) \right) = 0 \tag{52}$$

Newton Lorentz Coulomb

Newtonian Perfect Equilibrium

$$\frac{1}{c^2}\,\overline{\phi} \cdot \overline{\phi*} = \rho\ [\mathrm{kg/m^3}]$$

These results lead to the conclusion that the results of the experiments, published in 2021 "Operational Resource Theory of Imaginarity " in "Physical Review Letters" present strong evidence for the existence at sub-atomic level of the **electromagnetic GEONs** and the correctness of Wheeler's theory (Equation (50) and (51).

IV. Data Availability

All the Data and Calculations" have been published in the 'Open Source Framework(OSF)': https://osf.io/gbn4p/ DOI: 10.31219/osf.io/gbn4p. (https://doi.org/10.31219/osf.io/gbn4p). (Calculations in Mathematica 11.0)', p. 1–33). And the Download Page: https://quantumlight.science/

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